IO. BAPTISTAE PORTAE NEAPOLITANI

ELEMENTORVM CVRVILINEORVM LIBRI TRES.

In quibus altera Geometriæ parte reſtituta, agitur de CIRCVLI QVADRATVRA.

Ad Illuſtriſſimum Principem ac D.

D. FEDERICVM CAESIVM MONTIS CAELII MARCHION. II. &c. BARONEM ROMANVM.



ROMAE,
Apud Bartholomæum Zannettum. M. DC. X.

SVPERIORVM PERMISSV.

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Imprimatur ſi videbitur R. P. M. Sac. Pal. Apoſt.
Cæſar Fidelis Viceſg.

LIbri tres Elementorum Curuilineorum Periluſtris & Excellentiſſimi D. Ioannis Baptiſtæ Portæ Neapolitani, ex ordine Reuerendiſſimi P. Magiſtri F. Ludouici Yſtella Sacri Palatij Apoſtolici Magiſtri perlegi, eoſque cum nihil fidei, vel moribus aduerſum continere inuenerim typis dignos exiſtimaui. Romæ Die 20. Iulij 1610.

Antonius Butius Fauentinus Ciuis Romanus Philoſophiæ & Medicinæ Doctor.

Imprimatur. Fr. Damianus à Fonſeca magiſter, & Socius Reuerendiſſimi P. Magiſtri Fr. Ludouici Yſtella, ſacri Palatij Apoſtolici Magiſtri, Ordinis Prædicatorum.

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In Clariſſimum ac Doctiſſimum Virum

IO. BAPTIST. PORTAM NEAP. LYN.

& in librum de Circuli Quadrato.

Ioannis Demiſiani Cephallenienſis D. Philoſophi ac Theologi.

ΗΜΟΣ ἐοῖς ἐπέεσσι πολύχροα Δαίδαλα ΠΟΡΤΗΣ
  Φαίνει, τοῖς θαλέθει γαῖα πυκαζομένη.
Οὔατι θαμβαίνουσα βιαρκεῖ μῦθον ἀκούει,
  Καὶ σοφετέρης γ᾽ αἰεὶ φέγγεσιν ἀγλαΐης.
Ἀξυγέτου βάζοντος ἀπείρετα θαύματα πόντου,
  Σιγαλέη πόντος νηνεμίη γελάει.
Ἠέρος αἰγλήεντος ὅλην χύσιν αὐτῆς ἐνίσπει,
  Ἶσον ἀπαστράπτει δώμασιν οὐρανίων.
Αἰθέρος ἀστροχίτωνος ἀτειρέα νῶτα πιφαύσκει,
  Καὶ Πόλος ἠρεμέων οὐδ᾽ ἑτέρωσε θέει.
Κύκλα δὲ, καὶ Τετράγωνα, Τρίγωνά τε, Πρίσματα, Κώνους,
  Καὶ γραμμὰς μετρεῖ, κέντρα τε πυραμίδων.
Ὅσσα τε πὰρ Νείλου προχοῇς μητίσατο τέχνη
  Αἴα δαιτρεύων Ἠερίης ναέτης.
Αὐτίνυν Σοφίη πολυμήχανα δήνεα τίει,
  Ὕδωρ πρηῢ ῥέον Νεῖλος ἐρυκακέει.
Ἔδρακε γὰρ Τετράγωνα πάρος πολεμήϊα Κύκλοις,
  Ὅρκια συνθεσίης, καὶ φιλίης τάμνειν.
Ἔδρακε, καὶ θάμβησεν ὅπερ χρόνος ὀψὲ φαείνειν
  Ἤμελλε, ζαθέων ἠῢ τέκος πραπίδων.
Ἄλλον ἐγὼ Κρονίδην δώσω πολυΐδμονα ΠΟΡΤΗΝ,
  Ἣν ἡμῖν ἄλλην Παρθενόπην ὀπάσεις.

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FRANCISCI STELLVTI
FABRIANENSIS.
LYNCÆI.

Vidimus innumeras mutantem Protea formas;
Credite, nam veri Nuncia Fama canit.
Tortilis en Orbis ſpecies ſe vertit in omnes,
Et QVADRVM teretes efficit arte rotas.
Dicite Pierides quo tandem munere factum?
Aut noſtro, aut PORTÆ. visq. laborq. pares.

ILL.

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ILL.MO PRINCIPI AC D. D.
FEDERICO CAESIO
MONTIS CAELII
MARCHIONI II.

Io. Baptiſta Porta Neapolitanus. S.

CERTAMVS inter nos, Illuſtriſſime Vir,tu beneficijs, ego officijs; quibus ęquo animo vel vincar abs te, vel, ſi fieri poſſet, vincam te. Et ſanè grauis iſta contentio nullum vnquam finem habitura videtur. Summis me ornas laudibus, meos libellos plauſu, nedum honore proſequeris, &, quod caput eſt, iacentes aliquando, ac mox improborum impetu proterendos, erigis & defendis; quæ quidem merita ita in memoria inſederunt mea, vt mei ipſius potiùs, quam illorũ erga me magnitudinis obliuio capiat. Ego verò ſi titulos percenſere velim,quibus tuũ animum virtus cohoneſtauit,ſplendorem domus,quam

„ Bellipotens illuſtrat Auus,Tu fulcis, & ornas.

aliaq. ornamenta, quibus te natura mirificè cumulauit; & vires, & vita me deficeret. quid? ipſam Inuidiam ad maxima quæque, ac pulcherrima labefactanda natam, virtute ſuperaſti.

„Eſt

„ Eſt aliquod meriti ſpatium, quod nulla furentis
„ Inuidiæ menſura capit.

Sed non eſt animus in præſentia laudes enumerare tuas. maioris id molis eſt. leuiter, at amanter tetigiſſe ſatis; neque enim qui Cœleſtium Orbium ornatum in parua deſcribunt tabella, de illorum pulchritudine quicquam demunt; parua, vt ita dicam, ſed concinna magnitudo. Quid igitur mirum ſi certos fines, terminosq. huic ſuauiſsimæ concertationi non conſtituo? Non patitur mea in te obſeruantia Victoriam. Tu, quæ tua eſt magnanimitas, cedere neſcis. Eſto lis ſub te Iudice. Tu te vince

„ Inq. animis hominum pompa meliore triumpha.

meum certè quidem tibi deuinxiſti, ac deuiciſti. Non excitabo teſtes ex monumentis, quæ in manus peruenerunt Sapientum. Sit hic liber tuo inſignitus nomine, amoris, ac venerationis in te meæ pignus ſempiternum. Circulum quadrare conaturus rem ſcilicet aggreſſus in eruditorum identidem commemoratam comitijs, in Philoſophorum agitatam ſcholis, in Mathematicorum iactatam iudicijs. Multos in hoc Theoremate me labores exantlaſſe, curas, & cogitationes euigilaſſe meas, ac pertinaci induſtria deſudaſſe, non inficias iuerim. An verò modum quadrandi Circuli inuenerim, ſicq. præmium, & fructuum meorum cœperim laborũ, non facile ſtatuerim. Id ſaltem aſſecutus mihi videor. Latiſsimum apetuiſſe campum ad meliora vel inueſtiganda, vel inuenienda. Verecundè tamen dixerim, plurima nos excogitaſſe, multa in diſquiſitionem vocaſſe,

caſſe, ſuisq. examinaſſe ponderibus, quæ nemo vſque in hodiernum diem odoratus quidem eſt. Immo, vt id quod ſentio, aperiam, opus magnis viris tentatum, ac tandem deſperatum, aut inchoauimus, aut perfecimus. nihil tamen in tanto, ac tali negocio pro certo affirmarim, te, non aſſentiente, tuæ enim Παλλάδος ὑπὸ πτεροῖς ὄντας ἄζονται βροτοί. tuo iudicio, ac patrocinio fultus, non morabor τοῖς Γεφυρεισταῖς. Tenes, opinor, memoria, incomparabilis vir, Epheſiorum factum. Illi dum hoſtili vexarentur bello, de rei euentu conſuluerunt Oraculum. datum reſponſum, ſi Rempublicam ſartam tectam cuperent, ad Tutelaris Numinis Templum Vrbē alligarent; quo peracto, hoſtes in fugam verterunt, Epheſumq. obſidione, ac metu liberarunt. Multi iam cogitant noſtra obſidere inuenta, machinas admouent, ac penè labefactant: ſed meus Apollo dudum me commonefecit, vt me meaq. tui Genij vinculis obſtricta, aduerſariorum impetus reprimam, ac frangam. Tuere igitur, Heros, litterarum, ac litteratorum Cenſor, quæ tibi dicata ſunt, eo vultu, quo intuentium allicis animos. Habes à Philoſophia non minora clementiæ, quam iudicij præſidia, vt illa nouos hoſce foueas conatus, hoc vt defendas. Vale, tecumq. creſcat tuæ Gentis ſpes, Patriæ columen, litterarum decus, meæ Neapoleos amores, Italiæ gloria. Kal. Iulij M. DC. X.

AD LECTOREM PRAEFATIO.

NON immeritò, Candide Lector, admirari ſatis non poſſumus de viris quibuſdam omni doctrinæ genere cumulatis, qui, cum mathematicas tractationes ſibi aſſumpſerint, atque in ijs cum laude verſati, ſint, de illa parte, quæ curuas complectitur lineas, nihil ferè commentati, aut meditati ſint. In quadrando quidem certè Circulo (re ſcilicet æquè decantata, atque ardua) plerique ingenioſi viri deſudarunt, & elaborarunt rectè ne, an ſecus, ipſi viderint. Ego qui noui aliquid moliri, non aliorum labores veluti fucus ſurripere ſtudeo, eandem quidem ſubiui aleam. Sed vt legitimè & expeditius id præſtarem, multa ex Euclideis elementis ad propoſitum argumentum tranſtuli, ac plurimas confeci demonſtrationes, ex quibus, aliquas, quæ ad rem facere videntur ſe legi, easq. vti curuilinearum figurarum elementa propoſui. Hinc ad perdifficilè Theorema de quadrando Circulo, progreſſus ſum. quid vero effecerim in re multis circunfuſa tenebris, & in quà ſummorum virorum ingenia errare potius, quam hærere viſa ſunt, aliorum eſto iudicium. ſi perfectionem non ſum omnino aſſecutus, conatus certè, & adumbratio tanti Theorematis laudandus.

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IO. BAPT. PORTAE NEAPOLITANI ELEMENTORVM CVRVILINEORVM

Liber Primus.

DEFINITIONES.

PRIMA.

LINEA curua eſt, quæ inter ſua nō æquè fluit puncta, ſed facto ſinu flectitur.

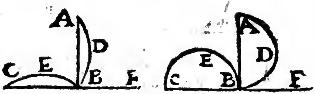


II.

Angulus flexilineus eſt flexarum linearum retuſio ſuo nutu ſibi coincidentium.



III.



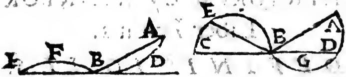
Angulus flexilineus rectus, qui rectilineo reſpondet.

Exempli cauſa ſit A.B.inſidens linea iacens FBC. vtrobiq. ſibi æquales conſtituens angulos ABF, ABC. ſitq. AB. ipſi B. C. æqualis & ipſi AB.hemicyclium circumſcribatur ADB.vel circuli portio, & ipſi BC. alter B. E C. vel æqualis circuli portio. Cyclogoni ergo DBA. CBE. ſunt æquales, & quanto angulus AD.B.F. maior eſt recto ipſo contingentiæ angulo DBF. tanto ABE. ſuperat ipſum ABC. altero contingentiæ angulo ABE.

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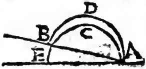
totus igitur ADBEC. toti ABC. recto æqualis, vt probauit Proclus in Eucl.

IIII.



Obtuſus curuilineus, qui obtuſo rectilineo fit quando à recto reſupinata in maiorem angulum abit.

Eodemq. modo angulū ADB. FE. flexilineum, rectilineo ABE, eſſe æquale flexilineus angulus FBE. eſt æqualis flexilineo DBG. nàm æquales ſunt circulorum portiones, ſi angulum DBG. abſtuleris, & repoſueris ſupra EB. erit rectilineus DBE. æqualis flexilineo DGBFE.



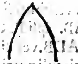
Sic etiā ſemicirculus ADB. æqualis eſt ACE, dematur portio communis ABC. remanet angulus CAD. æqualis rectilineo BAE.

V.



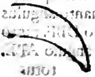
Xyſtroides angulus ſiue concauus quando vtrarumq. circumferentiarum caua extra fuerint, & intus ſe reſpiciens conuexitatibus ſuis.

VI.



Contra conuexus angulus quando circumferentiarum conuexa vtrinq; extra fuerint, & inter ſe ſuis finibus aſpexerint.

VII.



Angulus μηνοειδὴς ſiue lunaris, qui ex caua conuexavè circumferentia fuerit, vt conuexum vnius alterius conuexitatem aſpiciat.

Cyſ-

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VIII.

Cyſsoides Angulus ex hederæ folijs nomen indeptum. ex gibboſis, cauiſq. lineis conſtat ad punctum vnum conuenientibus, vndatim contra ſe diſcurrentibus, veluti Vndulatus.



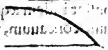
IX.

Mixtus angulus, qui ex rectis circuloſiſq. lineis componitur.



X.

Cyclogonus, qui à Caua, & recta circuli circumferentia conſtat.



XI.

Κερατοειδὴς ſiuè in cornua falcatus, quando rectæ opponitur conuexa noſtri contingentiæ vocant.



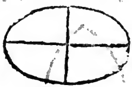
XII.

Figura vel anguloſa, vel agonia, agoniarum figurarum circulus princeps, lineæ partem, quæ ambitioſè circumuoluitur, & aream obambit concauum dicimus, quæ extorſum inuehitur conuexum.



XIII.

Sphærois ſiue Ellipſis ex ambienti linea in ſe recurſa deſcribitur vnius duæ diametri, longitudinis vna longior, latitudinis altera ad rectum in medio ſe ſecantes.



XIIII.

Vertex. ſiue corona eſt duorum circulorum concentricorum circumcurſus.



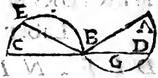
XV.

Anguloſarum figurarum metriſcus ſiue

totus igitur ADBEC. toti ABC. recto æqualis, vt probauit Proclus in Eucl.

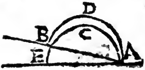
IIII.





Obtuſus curuilineus, qui obtuſo rectilineo fit quando à recto reſupinata in maiorem angulum abit.

Eodemq. modo angulũ ADB. FE. flexilineum, rectilineo ABE, eſſe æquale flexilineus angulus. FBE. eſt æqualis flexilineo DBG. nàm æquales ſunt circulorum portiones, ſi angulum DBG. abſtuleris, & repoſueris ſupra EB. erit rectilineus DBE. æqualis flexilineo DGBFE.



Sic etiã ſemicirculus ADB. æqualis eſt ACE, dematur portio communis ABC. remanet angulus CAD. æqualis rectilineo BAE.

V.



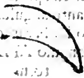
Xyſtroides angulus ſiue concauus quando vtrarumq. circumferentiarum caua extra fuerint, & intus ſe reſpiciens conuexitatibus ſuis.

VI.



Contra conuexus angulus quando circumferentiarum conuexa vtrinq; extra fuerint, & inter ſe ſuis finibus aſpexerint.

VII.



Angulus μηνοειδὴς ſiue lunaris, qui ex caua conuexavè circumferentia fuerit, vt conuexum vnius alterius conuexitatem aſpiciat.

Cyſ-

VIII.

Cyſsoides Angulus ex hederæ folijs nomen indeptum. ex gibboſis, & cauiſq. lineis conſtat ad punctum vnum conuenientibus, vndatim contra ſe diſcurrentibus: Veluti Vndulatus.



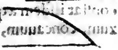
IX.

Mixtus angulus, qui ex rectis circuloſiſq. lineis componitur.



X.

Cyclogonius, qui à Caua, & recta circuli circumferentia conſtat.



XI.

Κερατοειδὴς. ſiuè in cornua falcatus, quando rectæ opponitur conuexa noſtri contingentiæ vocant.



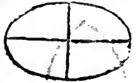
XII.

Figura vel anguloſa, vel agonia, agoniarum figurarum circulus princeps, lineæ partem, quæ ambitioſè circumuoluitur, & aream obambit concauum dicimus, quæ extorſum inuehitur conuexum.



XIII.

Sphærois ſiue Ellipſis ex ambienti linea in ſe recurſa deſcribitur vnius duæ diametri, longitudinis vna longior, latitudinis altera ad rectum in medio ſe ſecantes.



XIIII.

Vertex. ſiue corona eſt duorum circulorum concentricorum circumcurſus.



XV.

Anguloſarum figurarum metriſcus ſiue



lunula prior, eſtq. in eaſdem partes caua habentibus comprehenſa circumferentijs figura.

XVI.



Trilaterarum figurarum flexilinearum triangulum primum eſt, quod tribus conſtat ijſdem æqualibus circumferentijs circuli, idq. conuexum, concauum, vel mixtum.

XVII.



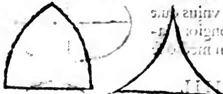
Iſoſcele triangulum curuilineum, quod duabus tãtum æqualibus circuli circumferentijs continetur; idq. etiam conuexum, vel concauum, vel mixtum.



XVIII.

Scalenum flexilineum eſt, quod tribus inæqualibus circuli circumferentijs clauditur, ijſq. cauis conuexis, & mixtis.

XIX.



Semicuruilinea triangula ſunt, quæ ex rectis, curuiſq. circumferentijs continentur.

XX.



Tricuſpidatum triangulum, ſiue acio idea quadrilaterum eſt triangulum, quod tres habet acutos angulos.

Inter

XXI.



Inter triangulares figuras πελεκοειδὴς. Figura eſt, quæ ſecuris vel bipennis formã habet.

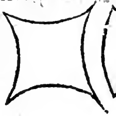
Eius Theocritus meminit. Nicandri Scholiaſtes ſutorium ſcalprum. Τὰ κυκλοτερῆ σιδήρεια, οἷς οἱ σκυτοτόμοι τέμνουσι καὶ ξύουσι τὰ δέρματα. Ideſt circularia ferramenta quibus pelles incidunt, & deradunt.

XXII.



Arbilones ex tribus circumferentijs compræhenſi; Horum meminit Pappus ſpatium illud inter circumferentias interiectum ἄρβηλον vocans.

XXIII.



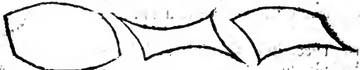
Quadrilaterarum quidum figurarum curuilinearum quadratum quidem flexilineũ eſt, quod rectis angulis, & æqualibus circumferentijs perſcribetur.

XXIIII.



Rhombus flexilinea æquilatera quidem, ſed non rectangula, aduerſos tamen angulos æquales habet, eorumq. aliquos concauos, conuexos, & mixtos.

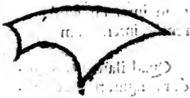
XXV.



Rhomboides vero neutrum horum habet neque laterum, neque angulorum æqualitatem, ſed contrarias circumferentias, & angulos æquales habet ſimiliter etiam concauus, conuexus, & mixtus.

XXVI.

Trapezoides curuilineum, quod quatuor inæqualia latera ex diuerſis circumferentijs habet.



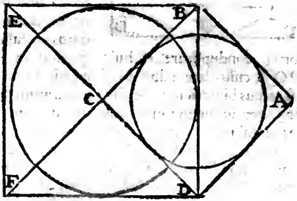
PRO-

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PROBL. I. PROP. I.

Datum circulum duplare.

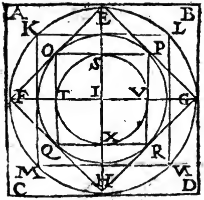
SIT datus circulus ABCD. cuius oportet duplum inueſtigare. Deſcribatur quadratum per 7. 4. Eucl. & ſit ABCD. ducto Diagonio BD. ſecundum datum BD. deſcribatur quadratum per 48. 1. Euclidis, & ſit BEDF. cui circulus inſcribatur per 6. 4. dico circulũ BDFE. eſſe dati duplum. Hanc



conſtructionem demonſtratione fulciendam rati ſumus. quoniam BCD. rectus eſt angulus proinde cum quadrata lateris BC. CD. æqualia ſint quadrato ex BD. ex 47. 1. ergo quadratum ex BD. duplum quadrati ABCD. ſed ex BD. deſcriptum quadratum eſt DBFE, ergo quadratum BDEF. duplum ipſius ABCD. ſed circulus ad circulum eandem rationem habet, quam quadratum inſcriptum, aut circumſcriptum, vt ex Euclidea demonſtratione ratum eſt duodecimi elementorum ſecunda, ergo circulum ABCD. duplauimus per circulum BEFD.

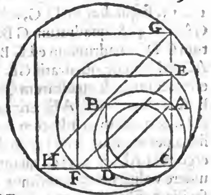
Plato ita quadratum duplat vt à Vitruuio annotatur. Dimidium quadrati BDC. eſt quarta pars quadrati BEF. ergo quadratum BEDF. duplum eſt ABCD.

Poſſu-



Poſſumus, & alio modo circulos duplare, ſi circa datum circulum quadratum induxeris, & poſt circa quadratum circulum, & circa circulum aliud quadratum eodem modo alios circulos ſemper duplicabis. Sed quo iuuenes rectius imaginari, & capere poſſint exemplo duximus declarandum. Eſto datus circulus STVX. quem oportet conduplicare, huic quadratum circumſtruemus O P Q R. cuius latera duabus diametris ſe ad centrum I, decuſſantibus bipartiemur, & circa quadratum O P Q R. circuculus alter deſignetur mox aliud quadratum. K. L. M. N. & alter circulus, ac demum aliud quadratum A B C D. quod poſtremum circulum K L M N. intercludat. His perſtructis aio aream inter circuli K L M N. finitionem concludam proximè ſepientis arctioris ſui circuli O P Q R. duplam eſſe, vt laxior poſtremi area eius, qui minimum intercludit quadrupla ſit, & ſic in infinitum duplare poſſumus cuius veritas hac demonſtratione repræſentabitur. Quoniam linea A B. bifariam diuiſa eſt in E, quadratum A B C D. quadruplum eſt ipſius A E, & ſic in quatuor quadrata æqualia A I, E G, F H, I D. & hæc à quatuor diagonijs bifariam diuiſa ſunt E F, F H, H G, G E, quatuor igitur triangula extrinſeca F A E, E B G, G D H, H C F. quatuor interioribus æqualia ſunt; ergo totum quadratum A B C D. quadrati E. F. G. H. duplum erit, eademq. ratione quadratum E F H G. ipſius O. P. Q. R. duplum erit, & primum A. B. C. D. huius quadruplum.

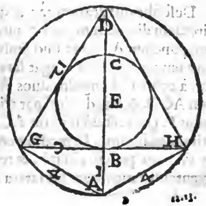
Anni-



Annitemur etiam per quadrata dupla ambientia idem rimari, & abſoluere. Eſto datus circulus A B C D, cui quadratum ABCD, circumſtruimus: mox ab oppoſitis angulis ducto diagonio AD & à puncto C, ſuperne verſus A, ſigna lineam eiuſdem longitudinis ipſius AD,& ſit C E, & ex parte inferiori ſit C F, mox trahe diagonium E F; & iterum quanta E F, figura in linea C G, & inferne linea CH, & id toties repetendum quoad ſatis videbitur. Sic quadratum ex GH, duplum eſt quadrati E F, & quadratum E F, duplum AD, & AD, duplum AC. Sed quod exprimit figura, demonſtremus. Quoniam quadratum AD, eſt æquale quadratis A C, C D, & AC, CD, latera æqualia ſunt, ergo quadratum ex A, duplum eſt quadrati AC,ſed AD,eſt æquale EC, ergo quadratum EC, eſt duplum AC. Sed quia E F, eſt æquale duobus quadratis EC, CF, & EC, C F, æqualia ſunt, ergo quadratum ex EF, duplum eſt EC. Eodem modo GH, duplum ipſius AC, & ſi idem varijs modis aſſequi poſſet, tanquam ſuffecturos reliquos cenſuimus miſſos facere.

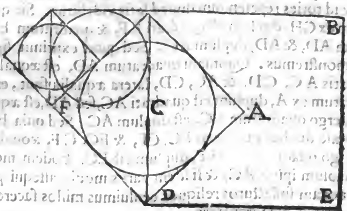
Libet non prætermittere, alium quadruplandi modũ.

Sic circulus B C, quem intendimus quadruplare circa quem æquilaterumtriangulum per tertiam quarti deſcribamus, & circa illud alium circulum per quintam eiuſdem quem quadruplum pronunciamus. Quoniam DG,tripla eſt ipſius GA. ex



ob

12. 13. ſi quadratum D G, erit duodecim partium talium GA, erit 4. & quadratum G B, erit talium 3. nam quadratum G D, quadruplum eſt G B, ſuæ dimidiæ, ſed quadratum AG, eſt æquale quadratis GB, BA, igitur ſi quadratum G A, erit talium 4. & quadratum GB, talium 3. erit quadratum B A. talium I, ſed A E, erit quatuor, quoniam eſt æqualis A G. & quando quadratum totum 4. eſt, & ſui pars 1. erit linea per medium diuiſa ergo AB. ipſius AE. dimidium erit, ergo tota AD. ipſius EB. quadrupla eſt. Si vero circulum diuidere voluerimus, poterimus conuerſa vti operatione, Et ſi facilia quidem ſint, quo tyrones iuuemus alium modum apponere non pigebit.



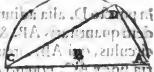
Deſcribe quadratum tantæ quantitatis quantæ duplarem circulum diuidẽdum fieri cupis, & ſit ABCDE. cuius medio fige punctum A. ſuper quo ambitioſa linea circumducatur, quæ omnia quadrati tangat latera, deinde annecte literas rectas à centro ad angulos duos AC. CD. & conſtitue triangulum ACD. & aliud priori par triangulum conſtitue cuius angulus F. erit rectus eſt igitur ACDF. ſecundum quadratum primi dimidium. In medio puncto huius diagonij CD, qui ſit G. pone pedem circini, & reliquo vago deſcribe circumferentiam tangentem ſui latera quadrati ACDF. & hoc mo-

do

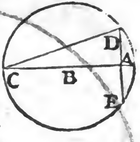
do in infinitum poteris circulos dimidiare. Demonſtratio ex ſuperiori pendet.

Datum circulum triplicem quintuplicem, & ſeptuplicem reddere.

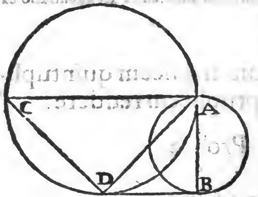
Prob. 2.



SIT dati circuli diameter AB. quem volumus triplare elongetur AB. in C. & ſit AB. æqualis BC. & fiat circulus ex diametro A C. & ſit AB. æqualis A D. quæ in circulo locetur per primam 4. Euclid. & ducatur DC. dico circulum ex DC. diametro circuli ex AB. tripli eſſe cuius demonſtratio ex 12. 13. lib. Eucl. pend.



Si vero quintuplare voluerimus ſit data diameter AB. circuli quintuplandi. Elongetur quantum AB. & ſit BC. circumducatur ei circulus A D C. in quo pentagonum æquilaterum inſcribatur per 9. 4. Eucl. & ſit linea ſubtendens duobus lateribus DC. pentagoni latus AC. dico quadratum DC. DE. ſimul iuncta quadrati AB. quintuplam eſſe. Demonſtrationem quære ex 12. 13. Euclidis.



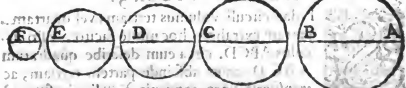
At is modus, vniuerſalior, & commodior viſus. Sit datus circulus A B. dimetiente deſcriptus, volo tergeminum reddere. Puncto igitur B. ipſius lineæ A B. ad rectos angulos adiungatur D B. paris quantitatis. Mox trahatur AD. dein ipſius lineæ DA. In puncto D. alia adiungatur DC. ad rectos angulos, & eiuſdem quantitatis AB, & ducatur CA, & dimetiente CA. fiat circulus, qui AB. circuli tergeminus erit. Quoniam potentia lineæ AC, potentia linearum AD. DC. ſibi vendicat, & AD. ipſas AB. BD. igitur AC. valet tres circulos, cuius ineſt A B. Quod ſi quintuplare, aut per alios impares numeros multiplicem reddere voluerimus: Addemus puncto C. lineam alteram ad pares angulos quantitatis AB. & erit quintupla ipſius AB.

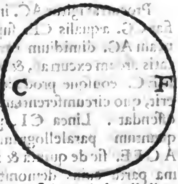


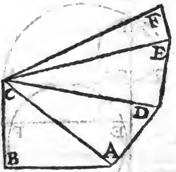
Poſſumus etiam ſi velimus alio modo idem exquirere.

Sta-

Statuatur circulus A B. C D. ſepties multiplicandus cui circumducatur quadratum, & latus eius producemus, illudque in octo partes diuidemus, cuius principium D. finis E. mox DE. per medium diuidatur in F. poſitoq. circini pede in F. & alio DF. circumducatur quouſq. ſemicirculum abſoluat DE. & latus C. B. quadrati producatur vltra B. in continuum, rectumq. ad arcum DE, & vbi eum contingit, illic ſcribe literam G. & ex CG. fiat quadratum CGHE. in quo circulus inſcribatur, qui continebit ſepties ipſum BACD. Quoniam CG. eſt media proportionalis inter EC. CD. igitur per 13. 6. Euclid. vt EC. prima ad tertiam CD. ita GH. quadratum ſecundæ ad BD. quadratum tertiæ per 20. 6. Eſt autem EC. per conſtructionem ſeptupla ipſius CD. igitur quadratum HC. ſeptuplum ipſius quadrati BD. quod probandum aſſumpſimus.







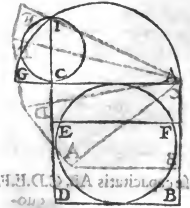
Sint poſiti quini circuli diuerſæ capacitatis AB. C.D.E.F.

quo-

quorum quantitates volumus ſingulari circulo comprehendere, quod ita propemodum faciendum exiſtimamus. Eſto enim circuli diameter AB. conſtituatur ad rectos angulos ei BC. mox ducatur linea ab A. ad C. & hæc dimetiens poteſt binos circulos AB. C. Porrò puncto A. lineæ AC. recta linea erigatur ad rectos angulos, quæ ſit AD. & à puncto D. trahatur linea D.C. & hæc dimetiens eſt capiens tres circulos AB, C. D. ipſi demum CD. recta linea ad rectos erigatur D E. quarti circuli dimetiens potens quatuor circulos. Poſtremo ei lineæ EC. ad rectos iterum excitetur quinti circuli EF. trahaturq. per FC. dimetiēs, capiēs iam cunctos circulos, & hoc modo omnes licet quotquot volueris comprehendere. Demonſtratio habetur ex penultima 1. libri Euclidis.

Ex dato circulo datam partem ſubtrahere. Probl. 3.

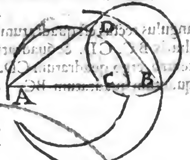
SI dati circuli volumus tertiam, vel quartam partem extrahere, hoc modo facito. Eſto circulus ABCD. circa eum deſcribe quadratum A B C D. cuius abſcinde partem tertiam, ac tranſuerſa linea conuenit à reliquis ſupernè diſtinguere FE.



Procurrat igitur AC. in G. & fiat CG. æqualis CE. ſupra lineam AG. dimidium rotunditatis arcum excurrat, & linea DEC. eouſque producenda erit, quo circumferentiam in I offendat. Linea CI. poteſt quantum paralellogrammum ACFE. ſic de quinta & ſeptima parte cuius demonſtratio ex vltima ſecūdi depēdet Eucl.

Datis

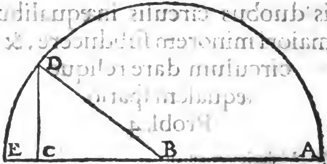
Datis duobus circulis inæqualibus à maiori minorem ſubducere, & circulum dare reliquo æqualem ſpatio.
Probl. 4.



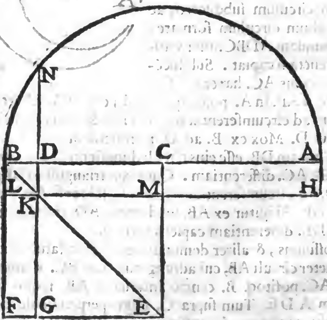
Subducitur etiam circulus minor à maiori, & circulus etiã formari poteſt, qui vtriuſque diferentiam capiat. Eſto maior circulus ABD. volo ab eo circulum ſubducere, ac mox alium circulum formare, qui lunulam ADBC. inter vtrũque relictam capiat. Subducẽdus circulus AC. hæreat in fine diametri AB. in A. poſitoq. circini pede in A. altero ad C. vagum ad circumferentiam traducito, & vbi eam incidit, ibi locetur D. Mox ex B. ad D. tranſuerſa ducatur linea DB. Dico lineam DB. eſſe eius circuli dimetientem capientem inter AB, AC. differentiam. Quoniam trianguli ADB. angulus D. ad circumferentiam rectus eſt, ſubtenſa AB. poteſt, vt AD. DB. Si igitur ex AB. ſubducatur AD. circulus, remanet alter DB. differentiam capiens vtriuſq.

Poſſumus, & aliter demonſtrare. Extendatur linea AB. diameter circuli AB, cui adiungatur linea BC. diameter circuli AC. poſitoq. B. centro interuallo AB. facito ſemicirculum ADE. Tum ſupra C, erigo perpendicularem CD. quouſq. tangatur circumferentia in puncto D. & connecto BD. Dico CD. eſſe queſiti circuli diametrum. Quoniam C.

angu-



angulus rectus eſt quadratum, ſubtenſæ BD. æquale eſt quadratis BC. CD. & quadratum BD. eſt æquale AB. quia ex centro, ergo quadratum CD. tanto minus eſt quadrato BD. quantum quadratum BC.



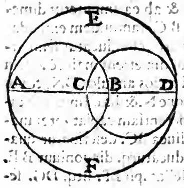
Quod ſi voles alio modo efficere hac ratione aſſequeris.

Sit

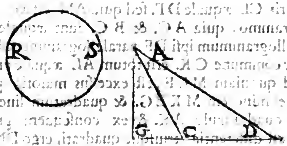
Sit dimetiens maioris circuli CB. & ab ea amputetur dimetiens minoris circuli CD, & linea B C tantundem extendatur ad A, & puncto C facto centro circumducatur ſemicirculus ANB. & à puncto D, vbi minor dimetiens maiorẽ abſcindit, erige ſuper tranſuerſam AB ad rectos angulos DN. & vbi DN periferiam ſecat ANB. iſtic pone N. & hæc linea erit dimetiens circuli inueniendi, qui differentiam capiat inter maiorem, & minorem circulum. Ex linea BC. deſcribatur quadratum per 46. p. E. & ſit CEBF. ducaturq. diagonium B E. & per D punctum deſcendat paralellas ipſi BF. ſitq. DG. ſecabitq. diagonium in K & per K ſignum excitetur alter paralellus ad AB, & ſit HMKL. & ex A ad H ducatur alter paralellus ipſi C M. Quoniam ſupplementum CK. ſupplemento KF. per 43. 1. eſt æquale, addatur commune quadratum D L. erit CL æquale D F. ſed quia AM eſt æquale MB paralellogrammo, quia A C. & B C. ſunt æquales, ergo AM paralellogrammum ipſi DF paralellogrammo eſt æquale, addatur commune C K. erit totum AL æquale gnomoni M L F. ſed quoniam M L F eſt exceſſus maioris quadrati CBEF. ſuper minorem M K E G. & quadratum lineæ D N eſt æquale quadrangulo A K. & ex conſequenti gnomoni MBF quæ eſt differentia vtriuſque quadrati; ergo DN circulus eſt differentia duorum inæqualium circulorum, quæ erat demonſtrandum.

Datis tribus circulis, duos à maiori, qui duobus circulis laxior ſit, ſubducere, & circulum dare reliquo ſpatio æqualem. Probl. 5.

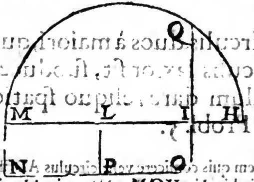
SIT amplius qualem quis conficere velit circulus ADEF. ſintq. pro arbitrio bini circuli A B. C D. quorum areæ



totam non contineant continentis amplitudinem, & ij vel in ſe ipſos flexi, vel mutuo interciſi, vt in exemplo, volo conſtringi circulū, qui reliquum ſpatium contineat, ſcilicet interceptum vacuum. Ex tribus AD. DC. BA. fiat triangulum ACD. quod obtuſum erit producaturq. alterutrius maioris circulatus, videlicet DC. eouſque ſit productionis meta, quouſque à trianguli ſupercilio, quod prædictæ lineæ incumbit, linea

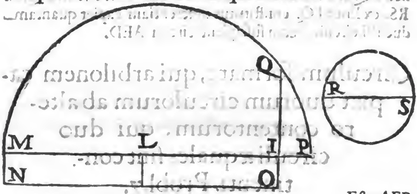


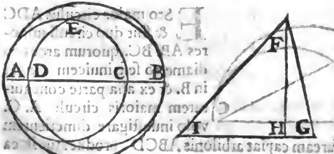
ad perpendiculum deſcendat, ſitq. AG. His perfectis extruatur paralellogrāmum, cuius productus latus ſit ex CD. geminata, & ſit ML. LI. breuius ex CG. ſitq. MNIO. & producatur MI. donec æquetur IQ. & ſit IH. & extremæ lineæ ora terminentur per circuitionis arcum MQH. elongeturq. OI. inferaturque coeun-



coeunti lineæ cum arcu litera Q. ſic, ex linea IQ. fiat circulus RS. capiens iam dictam differentiam. Quoniam angulus ACD. eſt obtuſus, quadratum lineæ AD maioris circuli ſuperat quadrata DC, CA, minorum circulorum per rectangulum compræhenſum ex DC. & CG. bis per 12. 2. Euclid. & ex his conſtitutum rectangulum MI, & diameter QI. capiet compræhenſam aream, ex qua circulus RS. quæſitam differentiam continebit.

Datis tribus circulis duos à maiori, qui duobus circulis anguſtior ſit, ſubducere, & circulum dare reliquo ſpatio deficienti æqualem. Probl. 6.



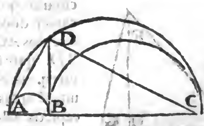


Eſto AEB. circulus, qui capiet duos circulos AC BD. quorũ vterq. cõcauitatẽ arcus capiẽtis cõtigat; ſuiſq.

C 2 ar-

arcis continentis areā excellant, veſtigandus eſt circulus, qui differentiam excellentis areæ excipiat. Fiat triangulum ex tribus lineis AB. AC. DB. per 22. 1. Euclid. & ſit GFI. qui erit acutus, cadat ex apice F. trianguli in ſubſtratam baſem GI. orthogonaliter linea FH. & vbi eam abſcindit, illic fige literam H. Porrò ex geminata baſe GI. & linea GH in ſe ductis, fiat paralellogrammum MO. & ſuperior linea MI procurrat quouſque ſit æqualis IO. & ſit P. Mox partire interuallum MP. per æqualia in D. & ex D centro deſcribe ſemicirculum, elongeturq. linea IO. quouſq. attingat arcum MP. in Q. & IQ dimetiens erit futuri circuli quæſitam differentiam capientis. Quoniam quadratum FI. minus eſt FG. GI. quadratis tantum, quantum rectangulum bis ſumptum ex linea IG. GH. per 13. 2. Euclid. quod erit NI. & linea IQ. erit dimetiens continens aream NI. circulus igitur RS. ex linea IQ. conſtitutus differentiam capiet quantam duo illi circuli aream ſuſcipient circuli AED.

Circulum formare, qui arbilonem capiat duorum circulorum ab altero contentorum, qui duo circuli æquales ſint continenti. Probl. 7.



Eſto maior circulus ADC & ſint duo circuli minores AB. BC. quorum arcus in diametro ſeſe inuicem tangāt in B. & ex alia parte concauitatem maioris circuli A. C. volo inueſtigare dimetientem circuli, qui aream capiat arbilonis ABCD. producatur linea

ex mutuo circulorum contactu B. donec rotundationis maioris circuli aream tetigerit BD. dico eam eſſe diametrum futuri circuli, qui arbilonis ABCD. aream continet. Hanc conſtructionem præſenti demonſtratione ſuffulciemus. Quoniam linea AC. ſecta eſt in puncto B. quadratum, quod fit ex A C. æquale eſt quadratis, quæ fiunt ex AB. BC. & paralelogrammo, quod bis fit ex CB. BA. ex imperio 4. 2. Euclid. Sed paralellogrammum ex CB. BA. eſt æquale quadrato DB. circulus ergo ex DB. eſt æquale arbiloni ABCD. quod quadratum ex DB. æquale ſit quadratis AB, BC. patet etiam ex 17. 6. Euclid. Vel quoniam circulus ex D C. æqualis eſt duobus circulis ex DB. BC. quia B. eſt angulus rectus, & circulus ex D A. circulis ex AB. B D. ergo circulus ex AC eſt æqualis duobus circulis A B. B C. & duobus circulis ex DB. qui in eo continentur, arbilon igitur ADCB. ex circulo D B. conſtat.

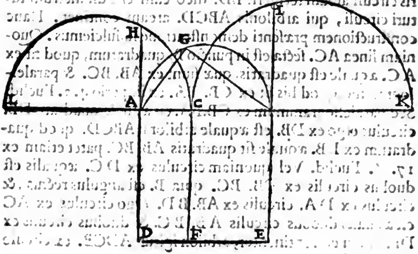
Corollarium.

EX hoc prouenit dato arbilone poſſe illico dari circulum ei æqualem, ſcilicet lineam erigendo ad duorum ſemicirculorum coniunctione ad circumferentiam.

Si diameter ſecetur vtcunque, circuli, qui fiunt ex tota, & ſingulis partibus continentur, æquales ſunt ei, qui à tota fit circulo.

Probl. 8.

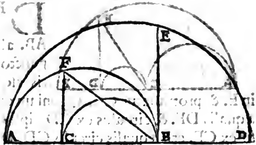
FIdat quadratum ex linea AB. & extendatur AB vſque ad K. & ſit æqualis AB. & ſupra CK fiat circulus C I K. & ex alia parte BA extendatur in L, & ſit æqualis AB. & ſuper



per CL. formetur circulus, & ſit CHL. & ſupra AB fiat alter circulus AGB. & ex C. extendatur parallellus ipſi AD. BE. & ſit FCG. extendaturq. DA ad H. & EB ad I. ducanturq. ex G. G A. G B. Quoniam quadrangulum ex KB. BC. eſt æquale quadrato BI. & quadrangulum ex LA. AC. eſt æquale quadrato A H. cohæreant ipſæ B I. AH. in circumferentia AGB. in puncto G. quia in circumferentia ad rectum angulum; ergo quadratum ex AB. duobus quadratis AG. GB. æquale erit, & ſic de circulis, vel aliter.

Quoniam per præcedentem diameter diuiſa bifariam in C. quadratum ex AC. & CB. & rectangulum bis contentum ex B A. A C. eſt æquale quadrato AB. ſed rectangulum bis contentum ex B A. A C. eſt æquale quadrato ex CG. ſed quadratum ex B C. C G. & quadratum ex A C. C G. ſunt æqualia quadratis ex A G. G B. & quadratam ex A G. G B. ſunt æqualia quadrato ex AB. ergo oſtendimus, quod intendebamus, & eſt ſecunda ſecundi Euclid.

Si diameter ſecetur vtrunq; circulus ex tota,& eius parte contentus æqualis erit circulo, qui ex partibus continetur, & eius quod ex prædicta parte fit circulus. Propoſ. 9.



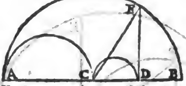
SIt diameter AB. ſecta vtcunq; in puncto C. dico circulum ex AB. BC. contentum æqualem eſſe circulo ex B C. C A. contento, & circulo CB.

Extendatur AB. in D. & ſit BD. æqualis ipſi BC. & ſuper ACBD. fiat circulus, & ſit AED. & ex puncto B. eleuetur perpendicularis vſque ad E. Idem fiat ex altera parte. Supra A C. & CB. duo circuli, & aſcendat ex C perpendicularis CF vſque ad ſemicirculum AFG, extendaturq. FB.

Quoniam quadrangulum, quod fit ex AB. BD. æquale eſt quadrato, quod fit ex BE. & quadrangulum, quod fit ex BA. AC. æquale quadrato ex CF. ſed quadratum ex FG. æquale eſt quadratis FC, CB. quia C. angulus eſt rectus ergo circulus ex AB. BC. quod eſt BF æquale eſt circulis ex CB. & qui fit ex BC. CA. & eſt 3. 2. Euclid.

Si

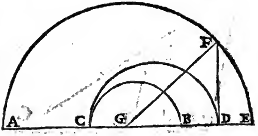
Si diameter ſecta fuerit in partes æquales,& inæquales circulus ex inæqualibus partibus contentus vna cum eo,qui fit ex linea,quæ inter ſectiones interijcitur æqualis eſt circulo, qui fit à dimidia. Prop. 10.



DEſcribatur circulus ex Diametro CA. alter ex AB. alter verò ex CD. ex D. puncto erigatur perpendicularis vſque ad circumferentiam in E. & protrahatur CE. Quoniam circulus ex AD. DB. eſt æqualis DE. & circulus ex CD. ipſi CD. ergo circulus, qui fit ex CE. erit æqualis circulis CD. DE. ſed CE. eſt æqualis CA. quia ex centro ad circumferentiam, ergo circulus ex duabus inæqualibus partibus compoſitus AD. DB. qui eſt DE. & circulus CD. vtrique æqualis eſt circulus ex dimidia CA. compoſitus, & eſt 5. 2. Euclid.

Si diameter bifariam ſecetur,eiq. in rectum adijciatur quædam recta linea, circulus ex tota diametro cum adiecta tanquam ex vno diametro,vna cum circulo dimidiæ æquales ſunt circulo ex dimidia., & adiecta tanquam ex vna diametro deſcripto. Prop. 11.

SIt diameter AB. ſecetur bifariam in C. & ei in longum adijciatur linea BD. dico circulus deſcriptus ex AD. DB. vna

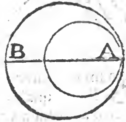


DB. vna cum circulo CB. æquales eſſe circulo,qui fit ex CD.

Lineæ AD. adijciatur DE, quæ ſit æqualis DB, & centro G interuallo GE deſcribatur circulus AFE. & ex puncto D linea ad rectum erigatur vſque donec circuli circumferentiam contingat, & ſit DF. & erit quadratum quadranguli AD. DE. & puncto D, linea DC ſecetur CB æqualis, & erit GD. & connectantur puncta GF. linea GD eſt æqualis CB. ex conſtructione. Quoniam linea AC. eſt æqualis lineæ CB. & CB ipſi GD. adijciatur ipſi AC. communis CG. & linea DE. eſt æqualis BD. ex conſtructione, ergo CD. ipſi GE. & angulus ad D. rectus eſt, valet ergo quadratum GF. quadrata GD. DF. ergo quadratum GF. valet quadratum CD, quod demonſtrandum propoſueramus, & eſt 6. 2. Euclid.

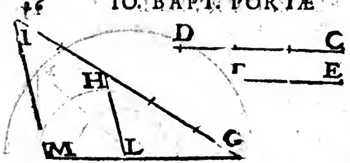
A dato circulo alium in datam proportionem abſcindere.

Prop. 12.



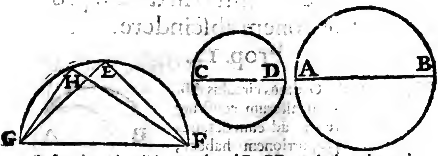
ESTO datus circulus AB. volo alterum conſtruere, vt ad eum datam proportionem habeat, ſitq. data proportio CD. ad EF. ſcilicet ſeſquialtera. Iungantur angulo binæ lineæ, quarum vna GH. ſit æqualis lineæ CD. protendanturq. quouſque HI ſit æqualis EF. Mox alteri lineæ æquetur dia-

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meter AB. quæ ſit GL. iungaturq. HL. & GL. extendatur, & à puncto I. lineæ H L. paralellus excitetur I M. dico LM. diametrum eſſe queſiti circuli A O. ſubſequialteri, & erit quarta linea proportionalis inuenta. Quoniam proportio GH. ad HI. eſt ſicut GL. ad LM. ex 12. 6. Eucl. & GH. ad HI. eſt ſeſquialtera, ergo GL. diameter ad AO. diametrum ſeſquialtera eſt.

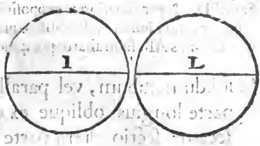
Ex duobus inæqualibus circulis duos æquales facere. Prop. 13.



Sint duo circuli inæquales AB. CD. volo hos duos circulos inæquales ad duos æquales reducere AB. DC. coniungo ad rectum angulum diametros, & ſint GHF. & connecto GF. tunc ſuper FG. facio ſemicirculum, q ui per H rectum angulum trãſibit, mox diuido circumferentiã in E. bifariam, &

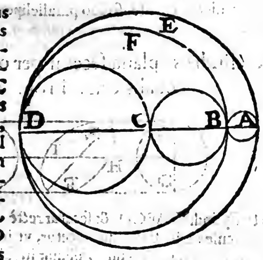
pro-

produc o GE. EF. dico duos circulos duarum dimetientium GE, EF. eſſe æquales duobus dimetientibus GH, HF. & proinde circulis I, L. Quoniam angulus H. eſt rectus, quia ad circumferentiam, ergo quadrata GH. HF. ſunt æqualia quadrato GF. & quadratis GE, EF. etiam æqualia quadrato GF. & quæ æqualia vni tertio æqualia inter ſe, ergo circuli I, L. ſunt æquales AB. CD.



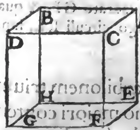
Circulum formare, qui capiat arbilonem trium minorum circulorum, ab imo maiori contentorum, qui tres circuli æquales ſint diametro continentis. Prop. 14.

ESTO circulus AED. cuius dimetiens AD. tribus circuli diametris intercidatur DC CB, BA. poſtulamus circulum formare, qui arbilonem, vel interceptam aream à maioris circuli cõcauitate, & minorum conuexitate contineat. Ex BD Diametro circulus



fiat BFD. & per ſuperiorem propoſitionem 7. arbilon BFDC capiatur, mox lunulæ AEDFBC quantitas cognoſcatur, à qua circulus AB. ſubtrahatur per 4. noſtram, & ſic de cæteris.

Si ſolidum cubum, vel parallelipedum altera parte longius oblique ex oppoſitis lateribus ſecetur ſectio altera parte longius erit. Propoſ. 15.



Esto ſolidus cubus ABCDEFGH & ſecetur à plano BDEF. oblique ex oppoſitis cubi lateribus BD. EF. dico BDEF. eſſe altera parte longius. Quia DG, GF, æqualis eſt. DF autem ſubiacens linea eſt æqualis duobus quadratis DG. GF. ergo longior BD. quæ ipſi DG æqualis eſt, idem dicendum de altera parte BH. HE. quia BE, maior eſt BH. HE. Igitur BDEF. altera parte longior eſt. Idem quoque dicendum de ſolido parallelipedo altera parte longiori.

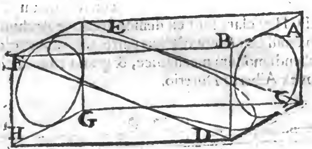
Si Cylindrus plana ſecetur per obliquum ſectio oualis erit. Prop. 16.



Sit Cylindrus ABCD. & ſecetur recte ABG. ſectio ACB. circulus erit, ſi oblique ſecetur, vt in IEHF. ſectio ſphærois erit ex ea quæ Serenus probauit in ſuis Cylindricis.

Si

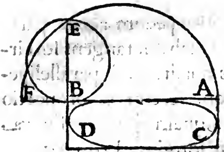
Si intra ſolidum parallelipedum altera parte longius cylindrus inſcribatut tangens ſui circuli baſis latera eius quadrati, & parallelipedum ſolidum obliquè ſecetur ea proportio erit circuli quadrato, quam ſphærois figura ad ſuum altera parte longius. Prop. 17.



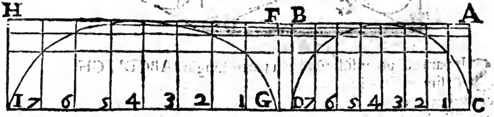
SIt parallelipedū ſolidū altera parte longius ABCDEFGH. & ſint cylindri in eo deſcripti baſes ABCD. EFGH. circuli in ea deſcripti ABCD. EFGH. & planum obliquè ſecans illud ſit CDEF. & ſphærois in eo deſcripta CDEF. dico ſphæroidem intra ſe deſcriptam eandem habere proportionem ad ſuam figuram altera parte longiorem, quam circulus ABCD. ad ſuum quadratum ABCD. cuius demonſtrationem omittimus: nam ex his, quæ Euclides in ſuorum elementorum, 12. & Archimedes in 31. præpoſitione deſcripſerunt, demonſtratur.

Data ſphæroide circulum eiuſdem areæ deſcribere. Prop. 18.

ESto data ſphærois ABCD, iubeo circulum eiuſdem ſpatij. Circa datam ſphæroidem quadrangulū circumſcribatur



batur ABCD. & latus AD prolongetur vſque ad F. vt BF. ſit æqualis BD. Et circa AF. ſemicirculus deſcribatur, elongeturq. BD. donec circumferentiam feriat, & ſit in puncto E. dico circulum conſcriptum circa BE. diametrum continere aream ſphæroidis ABCD. Hæc clara ſunt ex demonſtratione Archimedis libro de ſphæroidibus, & conoidibus parte 5.6.7. ſphæroidem idem deſcribendi modum mechanicè, & gratia commoditatis proponam ex Alberto Durerio.

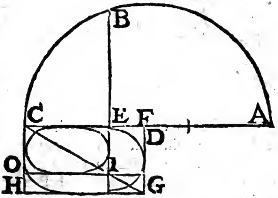


Deſcribe quadrangulum in duplo triplo, aut ſeſquialtero, & ſit in circulo ſupra AB. infernè CD. cuius latus CD. diuide in puncto E. per medium, ac poſito vno circini pede in puncto E. interuallo EC. ducatur per ſuperiorem partem vſque ad D. continget hic arcus lineam AB. deinde partire lineam CD. in octo æquales partes, & ex ſingulis diuiſionibus protrahe ſurſum parallelas in nuper deſcriptum arcum. Deinde fac iuxta quadrangulum ABCD. adhuc alium quadrangulum æqualis altitudinis, ſed longitudinis quantæ volueris cuius ſuperior linea FH. inferna vero GI. & ſeca id quoque in octo partes æquales, vt prius, poſtea producito ex ſingulis ſectionibus ſurſum lineas parallelas, deinde ex ſingulis inter ſectio-

ſectionibus prioris arcus, quæ per octo lineas parallelas factæ ſunt, parallelas tranſuerſales per omnes perpendiculares longioris quadranguli, & per ſectiones illas longiorem parallelorum arcum produc lineam arcualem de puncto in punctum incipiendo ab angulo G. & finiendo in I, vt vides.

Datam ſphæroidem duplare, vel quadruplare. Probl. *19*.

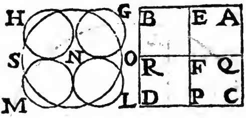
SIt duplandum quadrangulũ ECIO. quod idem eſt, ac duplanda ſphærois, quod intra illud circumſcripta eſt, & quadrangulum erit ſimile, ſimiliterq. poſitũ, quemadmodum, & ſphærois. Producatur



latus quadranguli EC. vſque ad A. & ſit AE. dupla ipſius AC. ac ipſius AC. medio D. poſito circini pede, DA interuallo, deſcribatur circulus ABC. producaturq. IE. vſque ad circumferentiam B. & erit EB latus vnum rectanguli deſcribendi. Reſcindatur igitur ex CA, linea CF, æquali EB, & ducatur diameter CI. deinde per F. ducatur parallela ipſi EI. quouſque occurrat diametro CI in G. & per G altera parallela ipſi FC producatur, quæ ſit GH compleaturq. parallelogrammum FH. erit igitur hoc parallelogrammum ipſi CI. ſimile, ſimiliterq. poſitum duplum. Quoniam AE. EB. EC. ſunt tres lineæ proportionales ex 13. 6. Euclid. erit vt AE, prima ad EC. tertiam, ita parallelogrammum FH. ex EF. ſecunda

cunda (nam CF ſumpta eſt æqualis EB.) ad parallelogrammum EO. ſupra tertiam EC. quod ſimile, ſimiliterq. deſcriptum.

Si circuli diameter bifariam ſecetur, & ex vna parte circulus fiat hic erit totius pars quarta. Prop. 20.

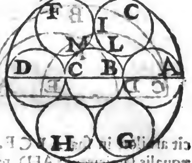


Rationes circulorum ſequuntur rationes quadratorum eis circumſcriptorum, vel inſcriptorum, & quẽadmodum, ſi quadrati diameter diuidatur quadratum ex vna parte, erit quarta parte totius, ita, & circulus; Exemplum latus AB. quadrati AD. diuidatur bifariam in E. dico quadratum ex AE. quod eſt AF. eſt AD quadrati pars quarta. Trahatur EP parallela, ipſi AC. & QR. ipſi AB, & erunt quatuor parallelogramma rectangula, & ſi aliter probari poſſet rationem recitabo apud Platonem in Memnone. Socrates enim puerum hoc modo docet. Sit bipedalis linea AB. dico ſuum quadratum eſſe quatuor pedum AQ erit vnius pedis, erunt dico quadrata QF. FR. ſit & altera pars CD duos pedes longa vnum alta C. erunt enim duo quadrata CF. FD. tota igitur quatuor erit pedum. Sit ergo circulus OILM. cuius diameter ONS. diuidatur bifariam in N. ex quantitate ON. quatuor circuli inſcribantur, dico quatuor hos circulos toti æquales eſſe. Ratio ex ſuperiori pendet: nam & circuli ſe habent ad quadrata, vt eorum diametri.

Cir-

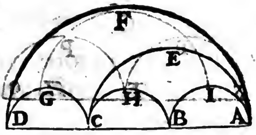
Circulorum vacua metiri, quando maior minores contineat. Prop. 21.

SIT magnus circulus AEFDHG, cuius diameter AD, diuidatur in tres partes, & in eo fiāt tres circuli AB, BC, CD, & ſupra duo aliȷ & duo infra inſcribantur; nam ſex circuli æquales intra vnum inſcribuntur ex 15. 4. Euclid. & ex præcedenti totus circulus nouem circulos cōtinebit: nam diameter trifariam diuiſa eſt, ſunt intus ſeptem contenti, ergo omnia vacua duo erunt circuli cuius 3. pars erit ſcalprum EIF. cum ſuo reſiduo ILM.



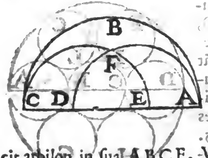
Arbilones per circulares figuras metiri. Prop. 22.

SIt arbilon primū AFDGCHBIA. inueſtigādum quot circulos capiet, qualis AB. Ex præcedēti ſemicirculus AFD nouem capiet ſemicirculos qualis AIB

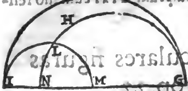


ſi ſubſtuleris AIB, B H C, C G D, erit arbilon reliquum ſex

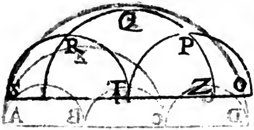
ſemicirculorum . Si quærimus arbilonem AFCHBI. erit ſemicirculus AEG quatuor ſemicirculorum qualis AIB, demptis duobus AIB, BHC, erit arbilon duorum ſemicirculorum. Si quærimus arbilonem AFDGCEA, erit ex iam dictis quatuor ſemicirculorum.



At ſi ſemicirculus maior ABC capiens duos ſemicirculos AED, EFC, vt docuimus in prima noſtri, dico arbilonem ABCFA eſſe æqualem duplato EFD, quod ex figura patet: nam quod replicatur in figura EFD deficit arbilon in ſua ABCF. Vel quarta pars dupli BEA. eſt æqualis ſemicirculo AFD. pars extera EFD eſt æqualis interiori corniculari angulo FBA.



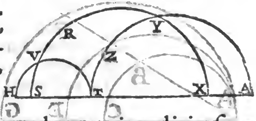
Idem eueniet in figura GHI: nam duo ſemicirculi GHLN, & MLI. per ſecundam noſtri capiũt aream continentis circuli GHI. Vnde duplatum MLN. eſt æqualie arbiloni GHLIHG.



Poteſt etiam euenire, vt arbilon medium PQRT eſt æquale duobus extrinſecis circuli partibus OPZ RS. ex ſuperiori ratione.

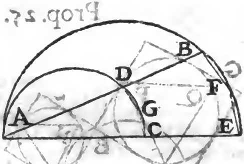
Idem

Idem eueniet in hac poſtrema, vt arbilon YZTVRY. ſit æquale duobus circuli extrinſecis partibus A Y X, S V H.



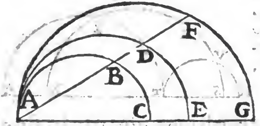
Si duo vel quamplures circuli in fine diametri ſe tangunt, à contactus autem puncto ducatur linea eos ſecans arcus ſecti inter ſe ſimiles erunt. Prop. 23.

Sint duo circuli ABE, ADC ſe mutuo tangentes in fine diametri A, & ducatur recta linea A D B, ſecans arcus ADC, in D, & ABE, in B, qui quidem arcus bifariam ſecentur, quia anguli in circulo oppoſiti per 22. 3. duo æquales rectis duobus in 2. ergo angulus B G C, & B F E æquales ſunt cum eodem BAC angulo iuncto.



Data circuli portione eam multiplicare, Prop. 24.

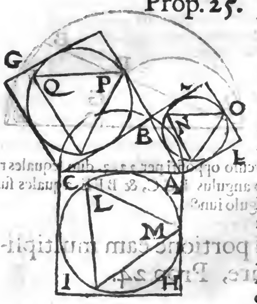
Sit data circuli portio A B, quam volo duplare & ſit eius circulus A B C, & ſit ſemicirculus A D E du-



plus dati. Per primam noſtri, linea A B trahatur longius in D, & ſi voluerimus quadruplare ſit circulus A F G quadruplus & linea A D in F extẽdatur, dico portionem DA ipſius BA duplam, & FA ipſius BA quadruplam cuius ratio pendet ex anteriori.

Ex duabus portionibus ſimilibus vnã ſimilem facere, vel ſubtrahere.

Prop. 25.

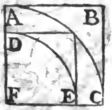


Sint duę inæquales circuli portiones PQ. ON. ſed ſimiles, & ſit vnaqueque tertia circuli pars per 25. 3. Euchi & ſint P Q G, O D N; circa quos deſcribantur quadrata B G, E B, vel eorum diametri, & iungantur ad rectum angulum ABC, & ſecundum AC deſcribatur quadratum, & in eo circulus MLI, & ſit ML latus æquilateri trianguli. Portio ML erit æqualis iam dictis dua-

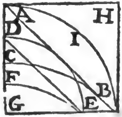
duabus portionibus per ea quæ in 3. Euclid. probantur. Vel ſi ex M L voluerimus portionem P Q ſubtrahere, è recto quadrato AC, ac ſupra AC ſemicirculo deſcripto, ponatur latus quadrati B C, & eius latus B A latus quadrati portionem ſimilem continentis. Et ſic poſſumus ex pluribus portionibus vnam facere, & omnia illa, quæ de integro circulo retulimus.

Datum ſemicuruilineum triangulum duplare, ſubducere, vel è duobus ſimilibus vnum facere.

Prop. 26.



SIt ſemicuruilineum triangulum D G E, quod volo duplare, & ſit circuli quarta pars F D E, fiat etiam circuli dupli pars, & ſit A G C, circa eam quartam etiam quadrati partem circumſcribo ABCF, dico triangulum ſemicuruilineum A B C G. duplum eſſe D G E, Quia quadratum A B C F duplum eſt D G F E inſcripta portio proportionalis erit. Et ſic ſubtrahere, & ex multis vnam facere poterimus ex ſupradictis.



Eodem modo triangulum D E G duplare poterimus, quod eſt æquale iam dicto: nam quadrati dimidium B H A eſt æquale B A G, ſi dematur portio B I A, æqualis BCG. remanet triangulum B A G. æquale BHA, iam dicto.

.O I

dicto. Vnde ſi voluerimus prædictum EDF ſemicuruili-
neum triangulum duplare, duplato quadrante
HABG, protractoq. diametro BA.
circulus duplus BIA, qui
erit BC deſcri-
batur
BC, & erit triangulum ABC du-
plum trianguli
EDF.

IO.

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IO. BAPT. PORTAE NEAPOLITANI ELEMENTORVM CVRVILINEORVM Liber Secundus.

AXIOMATA.

I.

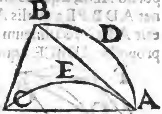
SI eidem addideris, quod prius dempſeris, quantitas æqualis erit.

II.

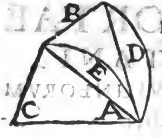
Si nota quantitas à nota ſubtrahatur, quæ remanet nota erit.

Triangulum ſemicuruilineum ex æqualibus, iiſdemq. circumferentijs compoſitum quadrare. Prop. 1.

ESto triangulum quodpiam ſemicuruilineum A D B C E, Aequalibus nimirum iiſdemq. circumferentijs ADB, AEC, & recta BC baſi conſtituta volo illud quadrare. Ducatur linea AB, & AC, aio aream trianguli ſemicuruilinei



ADBCE eſſe æqualem triangulo rectilineo A B C. Quoniam circumferentia A D B eſt æqualis portioni A E C, ablata A D B, repoſitaq. in A E C æquale remanet triangulum rectilineum A B C ſemicuruilineo per primum axioma noſtrum.

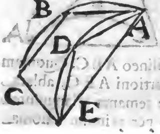


Vel fiat triangulum æquale rectilineum ABC, & ſit AFC ex 22.1. Euclid. erit ſemicuruilineum triangulum ADBCE, æquale triangulo ſemicuruilineo AECF dempta communi portione AEC remanet rectilineum AFC triangulo ſemicuruilineo æquale ADBCE.

Alter Caſus.

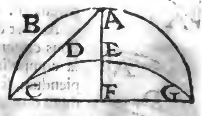


AT ſi triangulum ADBCE anguſtius erit, & portionis lineæ neutiquam intactas circumferentias relinquent, ſed per medium tranſibunt, eadem operatione idem aſſequi poterimus. Sed quo res dilucidior euadat, rem exemplo complectemur. Eſto triangulum ADBCE, & circumferentia ADB æqualis ſit AFC, trahanturq. rectæ lineæ AB, AC, & ſecet AB baſis ADB circumferentiam AEC, aio rectilineum ABC æqualem ſemicuruilineo ADBCE. Quoniam portio ADB, æqualis eſt AEC dempta communi AEF, remanet ADBFE æqualis AFC, apponatur vtrique areola FBC, erit ABC rectilineum triangulum ſemicuruilineo triangulo propoſito ADBCE æquale.



Vel ad eadem præſtanda poſſumus eaſdem circumferentias in plures partes diuidere, nempe binas, ternas, quaternas, vt ABC circumferentiam in AB, BC, & ADE in AD, DE. Vnde excluſæ partes AB, BC, incluſis AD, DE erit area rectilinea ABCEDA æqualis ſemicuruilineo ABCEDA.

Triangulum ſemicuruilineum ex varijs circumferentijs compoſitum quarum altera alterius dupla ſit quadrare. Prop. 2.

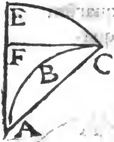


Esto triangulum ſemicuruilineum ABCDE cuius circumferentia EDC ſit circuli dupli ipſius ABC. Sed EDC ſit octaua pars circumferentiæ ſui circuli GEDC, circuli vero ABC quarta. Aio triangulum ſemicuruilineum ABCDE rectilineo inueſtigari poſſe parem. Rem ita moliemur. Completa circumferentia CE, ſit CEG, & coniungatur AC, mox portionem CEG diuidatur per medium, & ſit diuiſionis linea EF, dico triangulum AFC ſemicuruilineo triangulo parem eſſe. Quoniam tota portio ABC æqualis eſt dimidiæ ECF, id propterea dempta ABC portione repoſita EFC ſemicuruilineum ABCE, abijt in triangulum rectilineum ACF.



At ſi circulares lineæ magis cohærebunt, vt circumferentiarum baſes introrſum ſe ſecent, eadem erit operatio, & demonſtratio, vt in prima propoſitione. Productis lineis portionis AC, & ſemiportionis EFC triangulum rectilineum AEFC ſemicuruilineo par erit. Quoniam ſpatia ipſarum portionum ABC, EFC æqualia ſunt, ablata interiacente portione DC, quod reliquum eſt ABCD ipſi EDCF æquale erit, addita vtrique areola AED, erit totum triangulum rectilineum AEFC toti ſemicuruilineo ABCDE æquale, nam quanta pars ex

demptione abijt, tota ex repoſitione ſubſtituta eſt.



Vel poteſt tranſpoſitis lineis alio modo triangulum ſemicuruilineum conſtitui ſit circumferentia dupli C D E retro C B A ante, tunc ex puncto C. ſuper baſim A E cadat perpendicularis C F, & connectatur C A, & ſic triangulum ſemicuruilineum A B C D E rectilineo F C A parem iri. Ratio in ſuperiori.

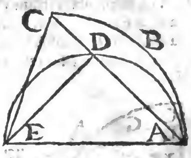


At ſivt diximus ex varijs, & inæqualibus circumferentijs orbiculata triangula compoſita erunt, tunc mente concipiendum, ſi circulus duplus alteri ſit, ſubdupli duæ circumferentiæ partes, vni dupli reſpondent, ſi quadrupli quatuor, & ſic deinceps. Eſto verbi gratia circuli dupli circumferentia E D C, & ſit octaua ſuæ circumferentiæ pars reſpondet duobus octauis ſubdupli circuli A B C. Diuidatur ambiens linea ABC bifariam in B, & trahatur A B, B C, E C, & erunt duæ AB, BC portiones, vni EC æquales, & ſic vna E D C, duas illas A B, B C abſumet. Vnde ſi triangulum ſemicuruilineum duabus octauis circumferentiæ partibus decreſcimus, AB, BC augemus vna EDC, & ſic par pari referemus.

Alter Caſus.

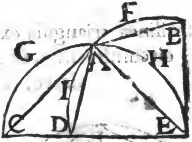
POteſt & aliter euenire; ſit triangulum ſemicuruilineum A B C E D, & ſit A B C quarta dupli circuli, & A D E ſemicirculus ſubdupli, docebimus quomodo poſſis rectilineũ triangulum æquale ſemicuruilineo facere. Trahantur ex puncto per medium circuli A D E vſque ad C, & ſit linea ADC, & linea DE. Erit triangulum ſemicuruilineum A B C E D

æquale



çquale rectilineo DCE. Quoniam portio A B C eſt dupla ipſius AD per 19. primi noſtri, & huic nempe portioni AD æqualis DE. dematur dimidia portio ABCD, addatur DE compar, remaneatq. communis areola DCEF, vtrique ſic enim rectilineum triangulum DCE æquale ſemicuruilineo ABCED, & ſic exceſſus vnius alterius defectu rependetur. Sic & in alijs notis circumferentijs quadruplis quintuplis eodem Methodo vti poteris.

Semicuruilinea triangula ad verticem conſtituta ex eiſdem, & æqualibus circumferentijs, vel ex æqualibus nota quadrare. Prop. 3.



SI duo ſemicirculoſa triangula ad verticem conſtituta ex eiſdem, & æqualibus circumferentijs fuerint ductis à vertice ad baſes rectis lineis, erunt rectangula circuloſis çqualia. Si primam huius libri leges non ſecus eſſe inuenies, quã diximus. Si acciderit, vt circumferentiæ eædem ad verticem ſint inæquales, ſed in id conueniant oportet, vt dextra interior ſiniſtræ exteriori æqualis ſit. Sint inæqualia triangula ſe inuicem decuſſantia BAE, ACD, ſegmenta ſint æqu alia vt BFA, AID, & EHA, AGC, tunc protractis rectis BA, AD,

 AE,

A E, A C, triangula rectilinea B A E, A D C, erunt circuloſis æqualia B F A H E, AGCDIA. Quoniam ſegmentum BFA æquale eſt AID. Si BFA ſeorſum expellimus,& AID ſua vice complectemur, ſic etiam reijcimus A G C reponimus E H A.



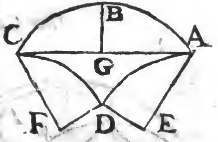
At ſi fuerint duo ſemicuruilinea triangula B E-A F D, & AGCIH conſtituta ad verticem A ex inæqualibus circũferentijs notis quarum DAC ſit circulus duplus ipſius BAI. Trahantur duæ lineæ perpendiculares ex A ad C I. & ſit A L, & A M ad B I. & binæ aliæ rectæ BA, AI, dico rectilinea triangula ALI, ABM, ſimul iuncta æqualia eſſe. Semicuruilineis BEAFD,CGAHI. Quoniam periferia DAC eſt circuli dupli quarta, & BAI ſubduplus ſemicirculus, duæ ſemiportiones A F D M, AGCL, abſument duas portiones BEA, A H L demptis igitur B E A, AGCL, repoſitiſq. AHI. AEBM, rectilinea triangula BAM, LAI, æquiualebunt ſemicuruilineis iam dictis.

Curuilinea triangula ex eiſdem & æqualibus circumferentijs, & ex varijs notis quadrare. Prop. 4.

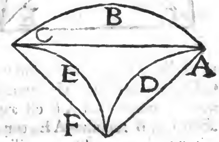


Esto curuilineum triãgulum ex tribus circumferentijs A B C, CD, D A, eiuſdem curuis, ſed ABC, dupla AD,DC conſtitutum quod quadrare intendimus. Dico protractis

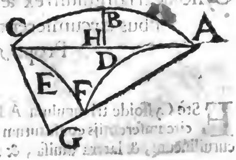
ctis æqualibus ſubtenſis AB, BC, CD, DA quadrilaterum rectilineum ABCD, eſſe æquale curuilineo ABCD. Quoniam demendo portiones AB, BC, addendoq. AD, DC, quæ ſimul æqualia ſunt voti compos fies, vel aliud dicimus.



Poterimus alio modo id aſſequi. Protrahatur linea AC, & binas AE, ED, & lineas CF, FD, vt ſemiportio AED ſit æqualis A B G, & DCF, & ipſi BGC, nam duæ portiones dimidiatæ ADE, CDF æquiualent vni integræ A B C. Vna hac dempta, his additis quod diximus eueniet.



Eodem modo curuilinea triangula ex inæqualibus circumferentijs, ſed altera alterius exempli cauſa ſit dupla. Sit curuilineum triangulum A B C E F D ex inæqualibus circumferentijs, ſed A B C dupla ſit ADF, & F E C ſubtenſis lineis A C, A F, F C erit quadratum nempe binæ portiones ADF, EFC æquipollent ſimplici A B C, vnde illa dempta, his additis triangulum rectilineum FAC æquipollet curuilineo propoſito.



Poteſt contingere, vt triangulum conſtituatur ex varijs circumferentijs, & inæqualibus, vt F E C ſit dimidia ipſius A B C, & ipſa ABC dupla ipſius A D F, ſic facta ſemiportio-

tione CFG, æquali BHC, & ſubtenſis A F portio A D F erit æqualis ABH. Vnde hac dempta, illis ſubditis triangulum rectilineum ACG erit æquale curuilineo ABCEFD.

Alter Caſus.



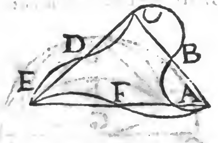
Esto curuilineũ triangulum ABCDFE propoſitum quadrãdum, & circumferentia circuli A B C ſit dupla E F D G C, diuidatur circumferentia E D C bifariam in D, & trahatur C D B erit ceratoide triãgulum B H C G D æquale portioni D G C per 19. primi noſtri. Vnde dempto B H G C D reponatur eius vice portio E F D æqualis D G C, & quia circumferentia A E eſt æqualis, & eadem ipſius A B, ablato A B repoſita A E, trapezuim rectilineum A B D E erit æquale propoſito curuilineo triangulo A B H C G D E F.

Cyſſoide triangulum ex æqualibus, & inæqualibus circumferentijs quadrare.

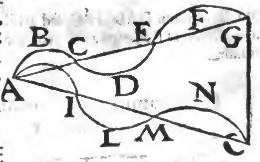
Prop. 5.

Esto Cyſſoide triangulum A F C ex tribus inæqualibus circumferentijs conſtitutum A B C, C D E, E F A curuilineum, & latera diuiſa, & æqualibus circumferentijs con-

conſtituta, vt A B, ſit æqualis B C, & C D, ipſi D E, & E F ipſi FA, vnde tractis lineis rectis AC, CE, E A, & demptis tribus circum ferentijs BC, DE, FA, & alijs tribus repoſitis A B, C D, E F, rectilineum triangulum A C E æquale eſt cyſſoidi ABCDEF.



Sit quoque ſemicyſſoide triangulum quadrandum A B C D E F G I L M N O, ex varijs circulorum circumferentijs, ſed tamen binis ſemper oppoſitis æqualibus conſtitutum videlicet G F E maioris circuli circumferentia, quam E D C, & E D C maior C B A, ſed tamen G F E æqualis O N M, & E D C, I L M, & C B A, A H I, ſi à puncto A ad baſim G O lineæ rectæ trahantur, totum aſſequeris, ratio pendet ex ſuperiori.

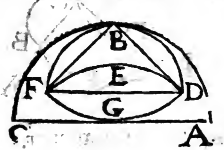


Arbilonem quadrare. Prop. 6.

ESto arbilon AH E C D B quadrandum, quia portio A H eſt dupla AB, & AB eſt æqualis ſemiportionis B G D, ergo ablata A B H, & repoſita



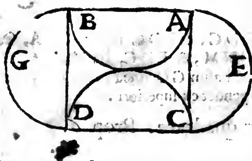
poſita BGD, & ablata HCE repoſita DCF, rectilineum GBHCF eſt æquale iam dicto arbiloni.



Poteſt & alio modo probari: ſemicirculus ABC eſt duplus ſemicirculi DBF, ergo vacuum ABDGFBC. eſt æquale ſemicirculo, dematur ex vtroq; portio DEF, DGF, ergo lunula DBFE eſt æqualis arbiloni DABGFBC, ſed arbilon eſt æquale triangulo rectilineo DEF, ergo arbilon dictum triangulo DBF, eſt æquale.

Quadratum curuilineum quadrare.

Prop. 8.



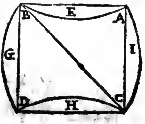
Eſto quadratum curuilineum AFBGDFCE. trahantur quatuor lineæ ex angulis AB, BD, DC, CA, dico quadratum rectilineum ABCD curuilineo iam dicto præſtabit. Quoniam ſunt quatuor ſemicirculi æquales inuicem, tollantur AEC, BGD, reponantur AFB, CFD, ſic rectilineum curuilineo æquale erit.

Alter Caſus.

Poteſt & quadratum aliter fieri, ex quatuor etiam rectis angulis, vt diximus ABC. Quoniam portiones

æqua-

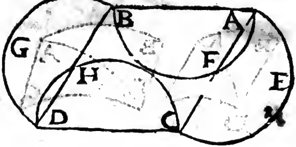
æquales ſunt, & ex æqualibus circulis, ablatis portionibus A I C, B G D, repoſitiſq. A E B, C H D rectilineum quadratum ABDC, curuilineo A E B G D H C I A æquipollebit.



Corollarium.

HInc patere poteſt quadratum curuilineum ex aduerſis, & conuerſis circumferentijs conſtitutum recta diameter bifariam ſecat, latus A B, lateri A C æquale eſt, & baſis B C communis vtrique, ergo triangulum C A B triangulo B D C æquale erit: igitur bifariam ſecat,& vtrumq. ex conuexo,& concauo æquali latere conſtat.

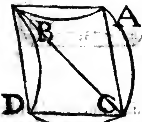
Rhombum curuilineum quadrare.
Prop. 8.



ET Rhombus curuilineus A F B G D H C E quadrabitur, ductis ex angulis rectis lineis AB, BD, DC, CA, nam demptis ſemicirculis A E C, D B G, repoſitiſq. AFB, C H D, demptiſq. portionibus HDAF, rectilineum Rhombum curuilineo æquabitur.

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Alter Caſus.

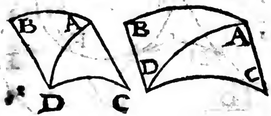


POteſt eſſe Rhombus alio modo ex æqualibus circumferentijs AB, BD, DC, CA, & quoniam portiones æquales ſunt, duabus demptis AC, CD, totidem repoſitis AB, BD erit rectilineo æqualis.

Corollarium.

EIuſmodi etiam Rhombos recta dimetiens æqualiter ſecabit; nam hinc inde duo æqualia triangula conſtituent.

Rhombos, ſeu Rhomboides ſemicuruilineos quadrare. Prop. *9*.

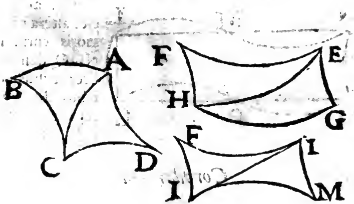


SEmicuruilineus Rhombus, & Rhomboides facilius quadrabitur: nam portione vna dempta, altera repoſita, æquales erunt curuilinei rectilineis.

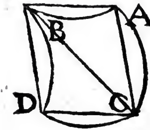
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Corollarium.

SEd in iſtis, qui ex iſoſcelibus triangulis ſemicuruilineis conſtituuntur curua diameter circumferentiæ æqualis, & eos bifariam ſecabit; nam in duo æqualia iſoſcelia triangula diuiduntur ſemicuruilinea, vt ABD, ADC.

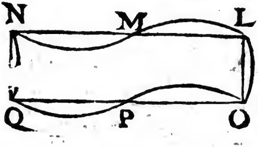


Poſſunt & alio modo Rhombos, & Rhomboides in Iſoſcelibus triangulis conſtitutos ex tribus conuerſis, & vna auerſa diametro per mediam diuidere, vt in Rhombo A B C D. Rhomboide E F G H; cum diameter A C, E H eos bifariam diuidat in duo Iſoſcelia æqualia A B C, A C D, & E H G, E H F, & in Rhomboide ex quatuor conuerſis conſtituto diameter recta etiam I L in duo ſemitriangula æqualia diuidit ex oppoſitis angulis ducta.



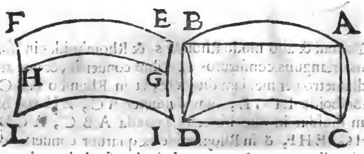
Altera parte curuilinea, & ſemicuruilinea quadrare. Prop. 10.

ALtera parte longiora quadrabis omnia, vt quadrata, duobus ſemper portionibus oppoſitis ablatis, & repoſitis, vt in ABCD.



Erit altera ſpecies altera parte longioris curuilinei LNOQ demptis ſcilicet tribus portionibus LM, PQ, OL, repoſitis MN, OP, QN, quadrabitur.

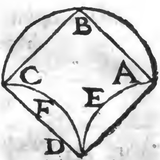
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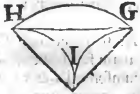
AT reliquas ſpecies diuides non dimetiente ex angulo ad angulum ducta, ſed per medium vtrinq. latera parallela, vt in EFIL, dimetiens GH.

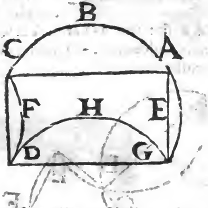
Pe-

Peleces quadrare. Prop. 11.



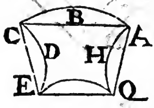
Poſſunt peleces multifariam variare ex varijs circulorum circumferentijs, & primo ex partibus, cuius partes circumferentiæ dimidij circuli A B C, aliæ duæ partes ex duabus quartis eiuſdem circuli A E D, D F C, vt demptis illis, his repoſitis, rectilineum quadratum peleci æquale erit.



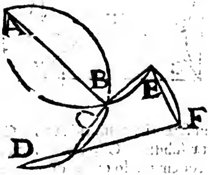


Poteſt & ex duplicis circumferentijs conſtitui, vt ſit G H quarta dupli, duæ verò quartæ ſubdupli G I, I H, quæ additæ rependent ablatum G H, eodem modo ex quadrupla eueniet. Pelecis ex inæqualibus, ſed eiſdem circumferentijs, & varijs, vt Pelecis G E A B C F D H quadranda portio A B C ſit æqualis G H D, & D F C, G E A, demantur A B C, reponantur G H D, D F C, & erit quadrilaterum rectilineum A C G D æquale ſupradictæ Peleci.

Trapezia curuilinea ex æqualibus, & inæqualibus circumferentijs conſtituta quadrare. Prop. 12.

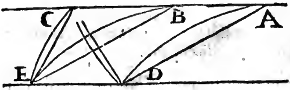


SInt Trapezia curuilinea ex quatuor, vel pluribus circumferentijs conſtituta, vel omnibus inæqualibus, vel tribus, aut duobus, dummodo inter eas ita conueniant, vt tres, duæ, aut plures poſſint, quantum vna, aut aliæ: nam ſit portio A B C tripla, & ſint tres æquales A H Q, Q F E, E D C, dematur maior, addantur tres minimæ, & coæquabitur rectilineum curuilineo.



At ſi Trapezium figuratum fuerit, vt ijſdē circumferentijs, & æqualibus conſtituatur, ſed cū alterū altero longius ſit, & quantum in altero deficit in altero ſuperſit, minus addatur ſuperfluo, & fiat æqua compenſatio. A B duæ portiones demantur, addantur duobus alijs B C, C D, & quia pars E F ſuperabit, deficit vero E B, huic addatur illius vice, ſic rectilineum B E F D C B curuilineo æquabitur.

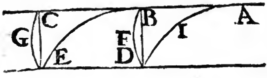
Triangulum Iſoſcele curuilineum,& parallelogrammum ſemicuruilineum in eadem baſi conſtituta,& eiſdem parallelis,parallelogram mum triangulum duplum erit, & rectilineis æqualia erunt. Prop. *13*.



SIt triangulum Iſoſcele ſemicuruilineũ DCE, & parallelogrammum ſemicuruilineum ABDE in eiſdem parallelis ACDE dico parallelogrammum in eadem baſi, & eiſdem circumferentijs conſtitutum eſſe triangulo duplum. Quoniam portio DC ipſi CE æqualis, dematur EC, addatur DC, erit triangulum rectilineum DCE curuilineo æquale. Et quia portio AD ipſi BE æqualis, dematur BE, addatur DA erit rectilineum parallelogrammum ABDE ſemicuruilineo æquale, ſed rectilineum ABDE triangulo DCE duplum eſt; quia in eadem baſi, & eiſdem parallelis conſtituta per 41. 1. Eucl. ergo parallelogrammum rectilineum curuilineo triangulo duplum.

Parallelogramma ſemicuruilinea in eadem baſi, & æquidiſtantibus circumferentijs conſtituta, & inter parallelas æqualia ſunt. Prop. *14*.

SInt duo parallelogramma BFDCGE, & AIDBHE in eadem baſi DE, & in eiſdem parallelis rectis AC, DE



D E conſtituta, dico inuicem eſſe æqualia: trahantur rectæ A D, B E, E C, quia portio BFD eſt ęqualis CGE dematur CGE, reponatur B F D, rectangulum parallelogrammum curuilineo æquale. Idem dicendum de altero parallelogrammo AIDB-H E curuilineo æquale eſt rectilineo ADBF, & quia parallelogramma rectilinea in eadem baſi, & eiſdem parallellis conſtituta ad inuicem ſunt æqualia per 36. 1. Euclid. Idem & de parallelogrammis curuilineis dicendum.

Parallelogramma curuilinea, & ſemicuruilinea cum æqualibus baſibus, & eiſdem circumferentijs, & eiſdem parallelis conſtituta inuicem ſunt æqualia.

Prop. 15.

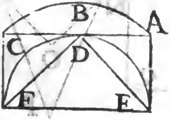


Sint duo parallelogramma ſemicuruilinea, & æquidiſtantibus circumferentijs AH, GB, & CF, DE, & æqualibus baſibus conſtituta FE, GH, & in eiſdem parallelis A D, H E dico eſſe inuicem æqualia, trahantur rectæ AH, BG, CF, DE, AF, B E, quia A H portio æqualis eſt B G, dempta AH repoſita BG, erit rectilineum AHBG curuilineo æquale, & idem de alio CFDE, ſed rectilineum AHBG curuilineo æquale, & idem de alio C F D E, ſed rectilineum CFDE in eadem baſi cum rectilineo ABFE, & ABFE in eadem cum A H B G, ergo inuicem æqualia per 26. 1. Euclid. ergo &c.

Paral-

Parallelogramma ſemicuruilinea in eiſdem parallelis conſtituta, & ex diuerſis circumferentijs videlicet duplis dari poſſunt rectilineis æqualia.

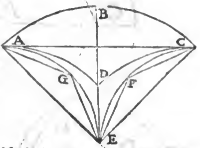
Prop. 16.



SIt parallelogrammum ſemicuruilineum E A B C F D E, & ſit portio CBA dupli, ſubdupli autem ſemicirculus F D, dico quadrari poſſe, trahatur linea C A, & F D, D E, quia duæ portiones ſubdupli F D, D E valent quantum vna ſubdupli CBA, dematur CBA, reponantur duæ ſubdupli F D, D E, rectilineum E A C F D valet quantum ſemicuruilineum.

Triangulum tricuſpidale quadrare.

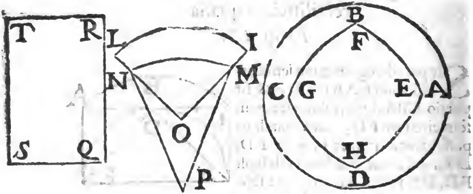
Prop. 17.



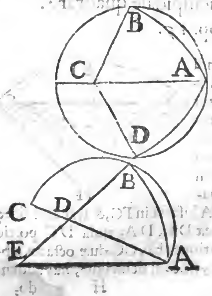
EX quarta noſtri ſecundi repræſentetur figura ABC F E G, à medio AC trahatur linea BE, & linea E B ſignetur in linea B E, linea CD ex eadem dupli circumferentia idem ex altera parte, dico triangulum tricuſpidale ADCFEGA quadrari Poſſe, diuidantur EC, EA bifariã in FG, & trahãtur lineę E F, F E, E G, G A, ſic, & lineæ D C, D A, quia D C portio eſt octaua pars ſui circuli, portiones EF, GC duæ octauæ ſubdupli æquipollent vnam dupli, ſic eam demendo, has adden-

do, trapezium EFCD rectilineum reſpondet curuilineo EFCD, idem de alia parte dicendum, & multifariam poteſt euenire.

Coronas quadrare. *Prop. 18.*

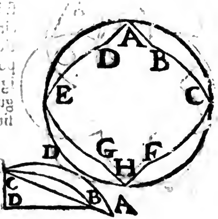


SIt quadranda Corona ABCDEFGH, pono eius quadrantem ILO, & ibi comparem octauã partem circuli dupli MNE, tollatur cõmune MNO, remanet tricuſpidale triangulũ quadrilaterũ MONP æquale quartę parti coronę IMLN, quæ æqualis ABEF, quadrupliceτur cuſpidale triangulũ, & erit ipſius area QRST æqualis coronæ propoſitę.

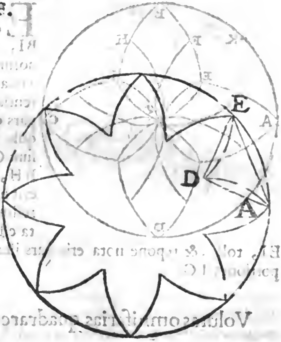


Eodem modo quarta pars ſemicirculi dupli ABE abſumit dimidium circulũ ABC ſubduplum, tollatur commune triangulũ ABD, remanet ADE triangulum rectilineum æquale Lunæ AB, & circuli BDC, quo duplato æquipollet coronę ABCD. Sit

Sit portio A D C B, æquipollet dimidiæ duplæ B C D, tollatur commune B C, remanet triãgulum B C D æquipollens ſemilunulæ A B C D, ſic quatuor triãgula B F G D abſumunt coronam C B A D E G H F.

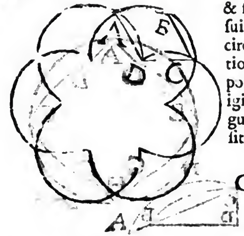


Per quartam noſtri ſecundi quadratur triangulum curuilineum A D E per rectilineum quadrangulum A B C D, ſex igitur eiuſmodi quadrãgula totam coronam abſorbent.



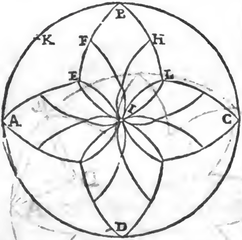
Eodem modo aliã coronæ ſpecies quadratur, & ſit eius pars A E D, cuius maior circulus A E ſit duplus A D, & ſit

H a



& ſit A D octaua pars ſui circuli, & A E ſui circuli, duæ igitur portiones A D, D E æquipollent vni maiori; octo igitur eiuſmodi triangula reſpondent propoſitæ coronæ.

Corona ſemiquadranda.



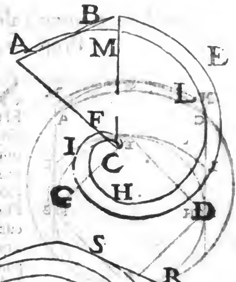
ESto circulus ABCD cuius quadrans ABI, triangulum ABE notum eſt, quia AE, E B æquales ſunt circumferentiæ A K, K B, reliqua pars quarta E F B H L I, quia B F, B H æquales ſunt G F, G H, tolle F B, B H, repone G F G H erit nota pars F B, H G, reliqua pars E F, I G nota eſt, quia æqualis FE, E I, tolle, & repone nota erit pars illa, remanent ergo 4. portiones I G.

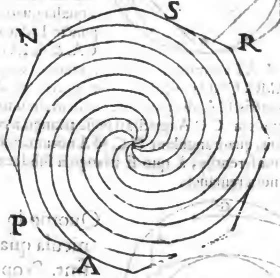
Volutas omnifarias quadrare. Prop. 19.

ESt voluta figuræ ſpecies in cocleæ modum ſinuata, cuius ambiens perpetuo flexu ducitur binis in ſe quodammo-

dammodo recuruis, & refractis lineis.

Propoſitum ergo ſit quadrare volutam BED GIFCHLMA ſcio hanc volutam octauam eſſe partem volutæ circuli NOPQRS, & omnes circuli volutæ non capiunt niſi octogoni aream, ergo vnaquæque octauam partem com-

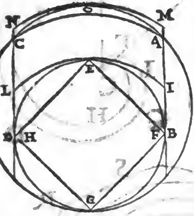




plectitur: vna igitur volutæ pars eſt BEDGIFCHLMA eſt octaua circuli pars, & octaua circuli pars eſt triangulum ABC, ergo tota propoſita voluta quadrata triangulo metitur. Poſſumus, & hoc modo cyſſoidem triangulum etiam quadrare, quod vidimus in 5. propoſitione huius.

Curui-

Curuilinea triangula aliqua quadrare. Prop. 20.



SIt circulus duplus AMN C, ſubduplus vero EFGH, ſeq. in puncto contingant G abſcindantur à duplo duæ portiones quadrati CN, AM, & à ſubduplo quatuor quadrati E F, FG, GH. Hæ remanent vacua CDEBA, BMG, GNH æqualia quadrato E F, G H puncto E ducatur parallela CA, & ſit L O quadrangulum C L A I notum eſt, remanent quatuor triangula L D E, EIB, AMO, CNO puncto dupli circumferentia ducatur MN, & ex punctis C A alia parallela eiuſdem circumferentiæ CA, dico lunulam C O A quadrari poſſe triangula N C O, OAM nota ſunt, quadrangulum N C M A notum, quia ex parallelis circumferentijs, à quo ſi triangula ſubducantur C O A lunula nota remanet.



Quomodo lunæ cornicula quadrari poſſint. Prop. 21.

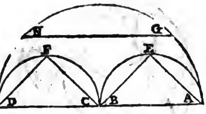
REmaneat ſuperior deſcriptio, & in ſemicirculo B G D deſcribatur dupli circumferentia BLID à punctis BD, lunula igitur BG-

BGADILB nota eſt, lunula parua nota eſt CGA, quadrangulum CLAI notum etiam ex anteriori, remanent ergo corniculi CBL, AID etiamnoti.

Trapezia multa curuilinea quadrare.

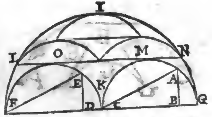
Prop. 22.

POſſumus quadrare Trapezium AGNDFCBEA, quia ſemicirculus AGND eſt quadruplus AEB per 20. noſtri, duo ſemicirculi AEB, CFD valent quantum arbilon AGNDCBA, dematur portio GN



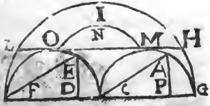
quarta circuli pars, & 4. portiones AE, EB, CF, FD, remanent duo triangula rectilinea AEB, CFD æqualia trapezio curuilineo iam dicto.

Eadem ratio erit in trigono. ſit trigoni portio LIN, & duo circuli FED, CAG, à quibus duæ portiones FE, CA, & duæ dimidiæ ED, AG, quæ vnam integrant, altera erit OIM, arbilon FLINGMCO

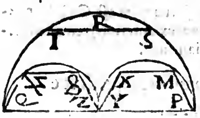


valet duos ſemicirculos, à quibus ſi tres dempſeris portiones, tres item ab arbilone, vacua FLO, OKM, MNG, NMI, IOL valent duo trigona FED, CAB.

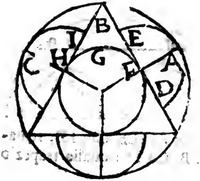
Eadem ratio erit in exagono, & trigono, nam in trigono in circulo GHILF, duo trigona APC, DEF, æquipollent vacuis GHM, MOC, OLF, HILONM, & in exagono PSRTQ,



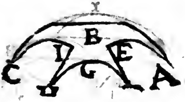
duo



duo ſemiexagona P M X Y, & Z 8 ☧ Q, valet vacuum P S T Q Y Z, in linea H L, tangens circumferentiam circulorum eſt latus trigoni æqui lateri per 12. 13. Eucl.



Circulus A B C, eſt quadruplus D A E, ergo pars tertia circuli A B C, quæ eſt A B C, eſt vnius circuli, & tertiæ partis, pars eius tertia eſt F G H, reliquum ergo erit corona A B C H G F dematur duo quadrantes circuli A E F, C H I, remanet vacuum AEFGHICBA, quantitatis dimidij circuli, & quia





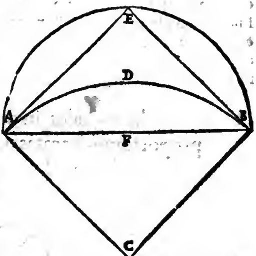
octaua pars circuli maioris, valet quatuor minoris dematur portio B ex maiori, & 4. 8. ex minori L M, M N, N O, O P, ergo rectilineam LMNOP, valet trapeſium AEFGHICBA.

IO. BAPT. PORTAE NEAPOLITANI

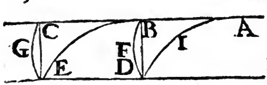
ELEMENTORVM CVRVILINEORVM Liber Tertius.

In quo de Circuli quadratura agitur.

Lunulam ex dupla, & ſubdupla proportione quadrare. Prop. 1.



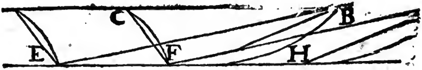
DEſcripto duplis circuli quadrãte his characteribus diſtinguatur ADBC, cuius ſubtenſam A B, ſcinde bifariam, & punctus ſciſſionis adamuſſim medius F characterẽ ſortiatur, in quo circini pede infixo ex FA interuallo circumducto ſemiambitum ſubdupli AEB ducito, aio triangulum rectilineum ABC interceptæ lunulæ areæ AEBD æqualem eſſe. In medio periferiæ ſubdupli E punctus inſtituendus, & ab vtraque circuli extremitate A B lineæ excurrant vſque ad E, ibique mutuo concurrant, quia dupli portio A D B F quarta ſui circuli pars valet quantum duæ ſubdupli portiones A E, E B, etiam ſui circuli pars quarta (per 20. primi noſtri) ideo



D E conſtituta, dico inuicem eſſe æqualia: trahantur rectæ A D, B E, E C, quia portio BFD eſt ęqualis CGE dematur CGE, reponatur B F D, rectangulum parallelogrammum curuilineo æquale. Idem dicendum de altero parallelogrammo AIDBHE curuilineo æquale eſt rectilineo ADBF, & quia parallelogramma rectilinea in eadem baſi, & eiſdem parallellis conſtituta ad inuicem ſunt æqualia per 36. 1. Euclid. Idem & de parallelogrammis curuilineis dicendum.

Parallelogramma curuilinea, & ſemicuruilinea cum æqualibus baſibus, & eiſdem circumferentijs, & eiſdem parallelis conſtituta inuicem ſunt æqualia.

Prop. 15.

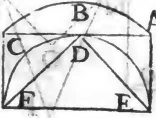


SInt duo parallelogramma ſemicuruilinea, & æquidiſtantibus circumferentijs AH, GB, & CF, DE, & æqualibus baſibus conſtituta FE, GH, & in eiſdem parallelis A D, H E dico eſſe inuicem æqualia, trahantur rectæ AH, BG, CF, DE, AF, B E, quia A H portio æqualis eſt B G, dempta AH repoſita BG, erit rectilineum AHBG curuilineo æquale, & idem de alio CFDE, ſed rectilineum AHBG curuilineo æquale, & idem de alio C F D E, ſed rectilineum CFDE in eadem baſi cum rectilineo ABFE, & ABFE in eadem cum A H B G, ergo inuicem æqualia per 26. 1. Euclid. ergo &c.

Paral-

Parallelogramma ſemicuruilinea in eiſdem parallelis conſtituta, & ex diuerſis circumferentijs videlicet duplis dari poſſunt rectilineis æqualia.

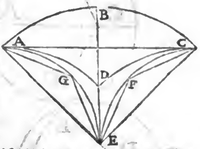
Prop. 16.



SIt parallelogrammum ſemicuruilineum E A B C F D E, & ſit portio CBA dupli, ſubdupli autem ſemicirculus F D, dico quadrari poſſe, trahatur linea C A, & F D, D E, quia duæ portiones ſubdupli FD, D E valent quantum vna ſubdupli CBA, dematur CBA, reponantur duæ ſubdupli F D, D E, rectilineum E A C F D valet quantum ſemicuruilineum.

Triangulum tricuſpidale quadrare.

Prop. 17.

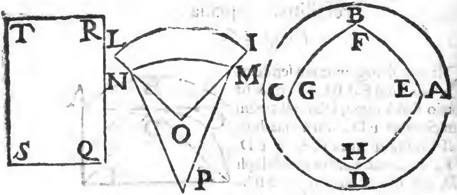


EX quarta noſtri ſecundi repræſentetur figura AB-C F E G, à medio AC trahatur linea BE, & linea EB ſignetur in linea BE, linea CD ex eadem dupli circumferentia idem ex altera parte, dico triangulum tricuſpidale ADCFEGA quadrari Poſſe, diuidantur EC, EA bifariã in FG, & trahãtur lineę EF, FE, EG, GA, ſic, & lineæ DC, DA, quia DC portio eſt octaua pars ſui circuli, portiones EF, GC duæ octauæ ſubdupli æquipollent vnam dupli, ſic eam demendo, has adden-

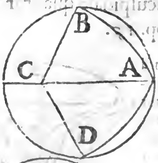
H do,

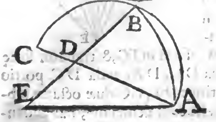
do, trapezium EFCD rectilineum reſpondet curuilineo EFCD, idem de alia parte dicendum, & multifariam poteſt euenire.

Coronas quadrare. *Prop.18.*

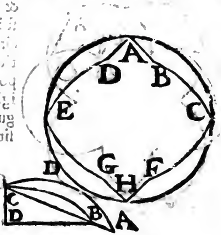


SIt quadranda Corona ABCDEFGH, pono eius quadrantem ILO, & ibi comparem octauā partem circuli dupli MNE, tollatur cōmune MNO, remanet tricuſpidale triangulū quadrilaterū MONP æquale quartę parti coronę IMLN, quæ æqualis ABEF, quadruplicetur cuſpidale triangulū, & erit ipſius area QRST æqualis coronæ propoſitę.

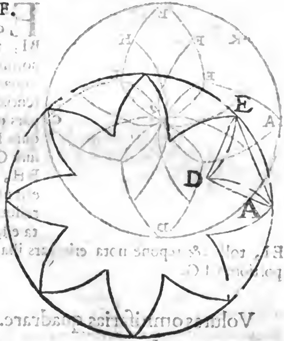




Eodem modo quarta pars ſemicirculi dupli ABE abſumit dimidium circuli ABC ſubduplum, tollatur cōmune triangulū ABD, remanet ADE triangulum rectilineum æquale Lunæ AB, & circuli BDC, quo duplato æquipollet coronę ABCD. Sit

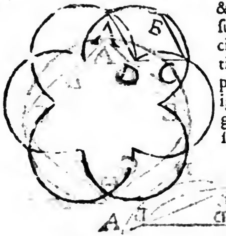


Sit portio ADCB, æquipollet dimidiæ duplæ BCD, tollatur commune BC, remanet triãgulum BCD æquipollens ſemilunulæ AB-CD, ſic quatuor triãgula BFGD abſumunt coronam CBADEGHF.



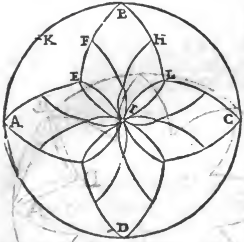
Per quartam noſtri ſecundi quadratur triangulum curuilineum ADE per rectilineum quadrangulum ABCD, ſex igitur eiuſmodi quãdrãgula totam coronam abſorbent.

Eodem modo aliã coronæ ſpecies quadratur, & ſit eius pars AED, cuius maior circulus AE ſit duplus AD,



& ſit A D octaua pars ſui circuli, & A E ſui circuli, duæ igitur portiones A D, D E æquipollent vni maiori; octo igitur eiuſmodi triangula reſpondent propoſitæ coronæ.

Corona ſemiquadranda.



EStò circulus ABCD cuius quadrans A-BI, triangulum ABE notum eſt; quia AE, E B æquales ſunt circumferentiæ A K; K B, reliqua pars quarta E F B H L I, quia B F, B H æquales ſunt G F, G H, tolle F B, B H, repone G F G H erit nota pars F B, H G, reliqua pars E F, I G nota eſt, quia æqualis FE, E I, tolle, & repone nota erit pars illa, remanent ergo 4. portiones I G.

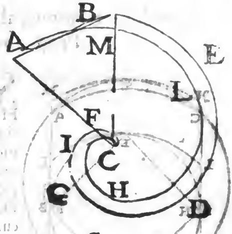
Volutas omnifarias quadrare. Prop. 19.

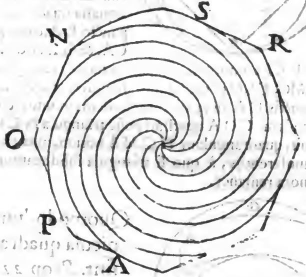
EST voluta figuræ ſpecies in cocleæ modum ſinuata, cuius ambiens perpetuò flexu ducitur binis in ſe quo-

dammo-

dammodo recuruis, & refractis lineis.

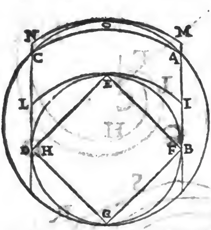
Propoſitum ergo ſit quadrare volutam BED GIFCHLMA ſcio hanc volutam octauam eſſe partem volutæ circuli NOPQRS, & omnes circuli volutæ non capiunt niſi octogoni aream, ergo vnaquæque octauam partem com-





plectitur: vna igitur volutæ pars eſt BEDGIFCHLMA eſt octaua circuli pars, & octaua circuli pars eſt triangulum ABC, ergo tota propoſita voluta quadrata triangulo metitur. Poſſumus, & hoc modo cyſſoidem triangulum etiam quadrare, quod vidimus in 5. propoſitione huius.

Curuilinea triangula aliqua quadrare. Prop. 20.



SIt circulus duplus AMN O, ſubduplus vero EFGH, ſeq. in puncto contingant G abſcindantur à duplo duæ portiones quadrati CN, AM, & à ſubduplo quatuor quadrati E F, FG, GH, HE remanent vacua CDEBA, BMG, GNH æqualia quadrato E F, G H puncto E ducatur parallela CA, & ſit L O quadrangulum C L A I notum eſt, remanent quatuor triangula L D E, EIB, AMO, CNO puncto dupli circumferentia ducatur MN, & ex punctis C A alia parallela eiuſdem circumferentiæ CA, dico lunulam C O A quadrari poſſe triangula N C O, OAM nota ſunt, quadrangulum N C M A notum, quia ex parallelis circumferentijs, à quo ſi triangula ſubducantur C O A lunula nota remanet.



Quomodo lunæ cornicula quadrari poſſint. *Prop.* 21.

REmaneat ſuperior deſcriptio, & in ſemicirculo B G D deſcribatur dupli circumferentia BLID à punctis BD, lunula igitur

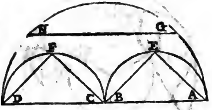
BG-

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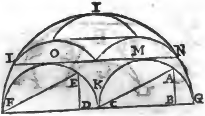
BGADILB nota eſt, lunula parua nota eſt CGA, quadrangulum CLAI notum etiam ex anteriori, remanent ergo corniculi CBL, AID etiam noti.

Trapezia multa curuilinea quadrare.

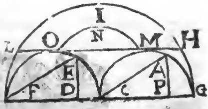
Prop. 22.



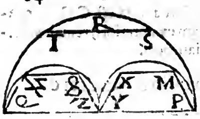
POſſumus quadrare Trapezium AGNDFCBEA, quia ſemicirculus AGND eſt quadruplus AEB per 20. noſtri, duo ſemicirculi AEB, CFD valent quantum arbilon AGNDCBA, dematur portio GN quarta circuli pars, & 4. portiones AE, EB, CF, FD, remanent duo triangula rectilinea AEB, CFD æqualia trapezio curuilineo iam dicto.



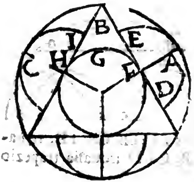
Eadem ratio erit in trigono. ſit trigoni portio LIN, & duo circuli FED, CAG, à quibus duæ portiones FE, CA, & duæ dimidiæ ED, AG, quæ vnam integrant, altera erit OIM, arbilon FLINGMCO valet duos ſemicirculos, à quibus ſi tres dempſeris portiones, tres item ab arbilone, vacua FLO, OKM, MNG, NMI, IOL valent duo trigona FED, CAB.



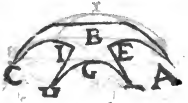
Eadem ratio erit in exagono, & trigono, nam in trigono in circulo GHILF, duo trigona APC, DEF, æquipollent vacuis GHM, MOC, OLF, HILONM, & in exagono PSRTQ, duo



duo ſemiexagona P M X Y, & Z 8 ⁊ Q, valet vacuum P S T Q Y Z, in linea H L, tangens circumferentiam circulorum eſt latus trigoni æqui lateri per 12.13.Eucl.



Circulus A B C, eſt quadruplus D A E, ergo pars tertia circuli A B C, quæ eſt A B C, eſt vnius circuli, & tertiæ partis, pars eius tertia eſt F G H, reliquum ergo erit corona A B C H G F dematur duo quadrantes circuli A E F, C H I, remanet vacuum AEFGHICBA, quantitatis dimidij circuli, & quia





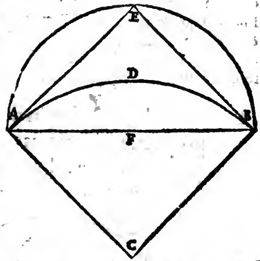
octaua pars circuli maioris, valet quatuor minoris dematur portio B ex maiori, & 4. 8. ex minori L M, M N, N O, O P, ergo rectilineam LMNOP, valet trapeſium AEFGHICBA.

IO. BAPT. PORTAE NEAPOLITANI

ELEMENTORVM CVRVILINEORVM Liber Tertius.

In quo de Circuli quadratura agitur.

Lunulam ex dupla, & ſubdupla proportione quadrare. Prop. 1.



DEſcripto duplis circuli quadrāte his characteribus diſtinguatur ADBC, cuius ſubtenſam AB, ſcinde bifariam, & punctus ſciſſionis adamuſſim medius F charactere ſortiatur, in quo circini pede infixo ex FA interuallo circumducto ſemiambitum ſubdupli AEB ducito, aio triangulum rectilineum ABC interceptæ lunulæ areæ AEBD æqualem eſſe. In medio periferiæ ſubdupli E punctus inſtituendus, & ab vtraque circuli extremitate A B lineæ excurrant vſque ad E, ibique mutuo concurrant, quia dupli portio ADBF quarta ſui circuli pars valet quantum duæ ſubdupli portiones AE, EB, etiam ſui circuli pars quarta (per 20. primi noſtri) ideo

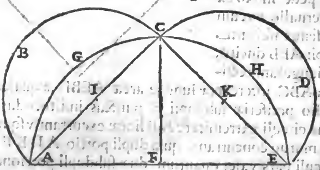
ſubductis portionibus A E, E B appoſita A D B (per primum axioma noſtri ſecundi libri) triangulum ABC valet quantum lunula AEBD, quod erat demonſtrandum.

Hyppocrates hoc aliter probat in primo Phyſicorum Ariſtotelis, quia quadrans dupli ADBC valet quantum ſemicirculus ſubdupli AEB, abſciſſa portione communi A D B, quæ inter vtrumque interiecta eſt, remanet trimetrum A B C æquale lunulæ AEBD, quadrandæ.

Conſectarium.

Ex hoc circumferentia dupli tranſibit ſemper per extremitates diametri ſubdupli, quod in alijs non euenit; quia angulus in ſemicirculo A E B rectus eſt (per 31. 3. Euclid.) quadrata A E, E B æqualia ſunt quadrato A B, & quadrantis dupli angulus ACB etiam rectus eſt, ergo quadrata AC, CB æqualia ſunt quadrato AB, ob id recta linea ſubtenſa quadrantis dupli eadem eſt cum diametro ſubdupli.

Duas lunulas æquales in dupla, & ſubdupla proportione exaratas ſeorſim quadrare. Prop. 2.



Eodem modo hoc commodiſſime abſoluemus. Eſto circulus

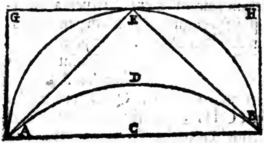
culus dupli A C E, & diametri extremitatibus A E binæ lineæ in medium circuli anfractum excurrant, & vbi mutuo contactu angulum efficiunt, illic G litera exaretur. Subtenſæ AC, CE in medio ductu præcidantur, præfixis literis I K, mox aſſumpto circino ad rem commodè proferendam, pede vno in I infixo ſubſiſtente I A interuallo linea circumducatur vſque donec ad alteram extremitatem C perueniat, inuariatoq. circini pede K puncto compari linearum perſcriptione circuli ſemianfractum deſignet A D E. Hoc peracto puncto F lineæ in plano iacentis perpendicularis extollatur, quæ in cuſpide curuaturam C contingat, aio lunulas A B C G, C D E H, æquales eſſe trimetro A C E. Quoniam circulus A C E duplex eſt A B C, C D E, ergo ſemicirculus A B C, C D E æquales ſunt ſemicirculo A C E, amputentur duæ communes portiones A G C, C H E, reſidua rectilinea triangula A C F, F C E æqualia ſunt lunulis ABGC, CDEH vel modo, quo ſupra præcepimus triangulum dupli A G C F æquale eſt ſemicirculo ABC, & triangulum F C H E æquale ſemicirculo C D E, ſubductis communibus portionibus A G C, C H E, triangulum A C E, eſt æquale duobus lunulis A B G C, C D E H.

Conſectarium.

Antequam vltra progrediar conſentaneum duxi adnotandum angulum rectum A C E bifariam diſſectum in C ex (per 8. 6. Euclid.) E C, ad C A rationem habet vt E F ad F A, & ſic triangulum C F E ad triangulum C A F (per primum 6. Euclid.) & quia æqualia ſunt, lunulæ quoque æquales ſunt.

Vacans ſpacium, quod intra figuras omnes notas interuenerit quadrare.

Prop. 3.

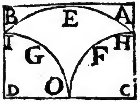


RECta linea A B dirigenda eſt, & ab eius vmbilici medio puncto C ſemiorbis circumducendus eſt A E B, completo ſemiorbi A C B, lunula complenda eſt more A E B D, mox laterales lineæ erigendæ ab extremitatibus A B ſunt, & medio eius puncto E ſuperior linea exaretur ipſi A B æquidiſtant,vt vltro, citroq. ſemicirculum tangant, etiam hiſce lateribus parallelogrammum exprimant AGHB, demum ab extremitatibus A B, medio puncto E tranſuerſas lineas ſortiatur A E, E B. Quoniam parallelogrammum GABH ſemicirculum continet, & eſt ſui quadrati dimidium, ſemicirculus lunulam continet AEBD, & lunula AEBD ſuo triangulo AEB æqualis eſt, & triangulum AEB ſui parallelogrammi dimidium eſt, ergo interceptæ areolæ AGE, EHB, ADB, quæ lunulam A B ambiunt, æquales ſunt ipſi lunulæ areolæ, ergo ſui amplexantis parallelogrammi dimidium ſunt.

Conſectarium.

EX hoc perſpicuum eſt lunulam ſui quadrati partem eſſe quartam; nam ſi lunula ſui obſepientis parallelogrammi dimidium eſt, & parallelogrammum ſui quadrati dimi-

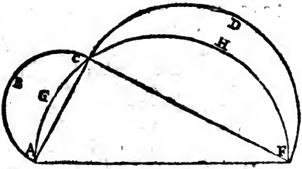
dimidium; igitur lunula ſui quadrati dimidium erit.



Vel ſi cuiuſcunque figuræ notæ vacua quadrare velimus modo ſupra cognito nota figura circumclaudatur, quam ſi à nota ſubtrahes, optato potieris ex ſecundo axiomate (ſecundi noſtri) ſit gratia exempli pelecis HEIGOF ſæpienda ſuo parallelogrammo ABCD, quam ab ipſo ſeduces, ſic incluſæ areæ HAE, EBI, IGOD, OCHF reſiduum innoteſcet.

Duas quaſcunque lunulas inæquales in ſemicirculo ſitas ſimul quadrare.

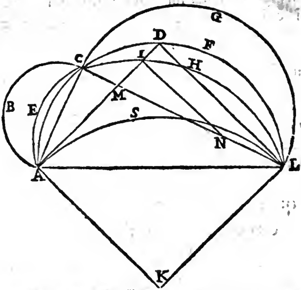
Prop. 4.



Triangulum rectilineum inſemicirculari linea definiri debet, quod tribus notis diſtinximus ACE, ſupra eius latera ſemicirculi incubabunt ABC, CDE, quibus congruens area adinuenienda eſt, inquam angulus ACE in ſemicirculo rectus eſt, & bini ſemicirculi ABC, CDE æquales ſunt ſemicirculo AGCHE ex eis, quæ ſupra habita ſunt, reiectis communibus portionibus AGC, CHE relictæ ſemilunulæ ABCG, CDEH reſiduo triangulo ACE rectilineo æquiparantur.

At

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At ſi perfectæ fuerint lunulæ quocunq. modo inæquales, ſemper orthogonio à circuli medio ad ſubſtratam baſim deducendo æquales erunt. ſit iacens linea A L, ſupra ſemicirculus ſtruatur ACFL, ab extremitatibus A L orthogonium quodcunque ſtruatur triangulum A C L, cuius laterum ductum rectum angulum in C conſtituant ſupra latera AC,CL, Ambientes ſemicirculi conſurgant ABC, CGL, & in eis perfectæ lunulæ ex more deſignentur A B C E, CGLH. His peractis in verticis ſinuoſæ lineæ puncto D ab diametri extremitatibus A L duo latera conſurgant, vt orthogonium triangulum ADL conſtituant, aio dictas perfectas lunulas ABCE, CGLH, in circulo A C D F L deſcriptum ſemper dicto triangulo ADL æquales eſſe: Quoniam lunulæ A B C E, C G L H æquales ſunt triangulo orthgonio A D L ſemper ex ſecunda huius.

Poteſt & alia probandi ratio ſuſcipi. Triangulo A D L ſuper lineam AL deſcripto, aliud triangulum æquale infra lineam AL deſignetur, & ſit ALK, & puncto K circini pede

fixo,

fixo, altero vago in A collocato ſinuoſa linea ducatur vſque ad L. Quoniam duæ lunulæ perfectæ ABCE, CGLH æquales ſunt vni lunulæ ASL (ex prima huius) & lunula ACDFLS eſt æqualis triangulo AKL, quod idem eſt cum triangulo ADL.

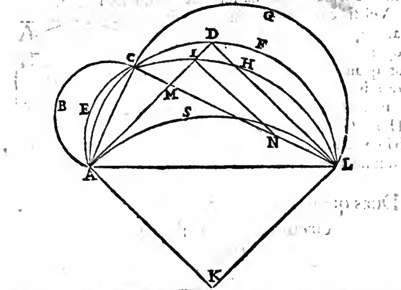
Conſectarium.

EX hoc animaduertendum imperfectæ lunulæ quanto magis à ſemicirculi vertice declinant, tanto minores fieri, vt in triangulo ACL videre eſt, quod triangulo ADL minus eſt, & qui defectum conſpicere quæſierit, triangulum ACM à triangulo MDL ſubducat hoc modo, à linea MD, linea MC præcidat, & à linea ML lineam MA obtruncet, & lineam MN ducat, triangulum MIN æquale erit ACM, reliquum triangulum IDN erit quantitas lunulæ IDML, dempla lunula AEC.

Vacua inter lunulas intermiſſa quadrare.

Prop. 5.

AT vero interuenientia vacua circa lunulas ſi quadrare quæſieris, ita quadrabis. Eſto minor lunula ABCG, maior CDLH imperamus ſemicuruilinea triangula inania inter illas, in rectilineas figuras reddere ſcilicet CPD, DQL, CHL, ARB, BSC, AGC, circumſcribantur parallelogrammata tangentia earum ambientes lineas PQCL, ARSC, & fiat alterum parallelogrammum ex binis AL, TV, & ſit ALTV, & fiat triangulum ACL æquale AFL (per 31. primi Euclid.) quibus ita diſpoſitis inquam vacuum circa ATVLF æquale eſſe imperatis vacuis.



At ſi perfectæ fuerint lunulæ quocunq. modo inæquales, ſemper orthogonio à circuli medio ad ſubſtratam baſim deducendo æquales erunt. ſit iacens linea A L, ſupra ſemicirculus ſtruatur ACFL, ab extremitatibus A L orthogonium quodcunque ſtruatur triangulum A C L, cuius laterum ductum rectum angulum in C conſtituant ſupra latera AC,CL, Ambientes ſemicirculi conſurgant ABC, CGL, & in eis perfectæ lunulæ ex more deſignentur A B C E, CGLH. His peractis in verticis ſinuoſæ lineæ puncto D ab diametri extremitatibus A L duo latera conſurgant, vt orthogonium triangulum ADL conſtituant, aio dictas perfectas lunulas ABCE, CGLH, in circulo A C D F L deſcriptum ſemper dicto triangulo ADL æquales eſſe: Quoniam lunulæ A B C E, C G L H æquales ſunt triangulo orthgonio A D L ſemper ex ſecunda huius.

Poteſt & alia probandi ratio ſuſcipi. Triangulo A D L ſuper lineam AL deſcripto, aliud triangulum æquale infra lineam AL deſignetur, & ſit ALK, & puncto K circini pede fixo,

fixo, altero vago in A collocato ſinuoſa linea ducatur vſque ad L. Quoniam duæ lunulæ perfectæ ABCE, CGLH æquales ſunt vni lunulæ ASL (ex prima huius) & lunula ACDFLS eſt æqualis triangulo AKL, quod idem eſt cum triangulo ADL.

Conſectarium.

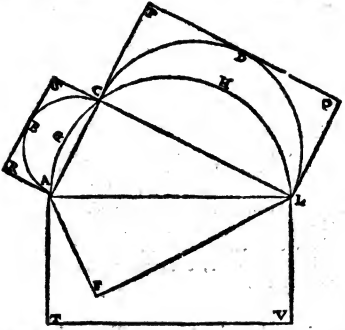
EX hoc animaduertendum imperfectæ lunulæ quanto magis à ſemicirculi vertice declinant, tanto minores fieri, vt in triangulo ACL videre eſt, quod triangulo ADL minus eſt, & qui defectum conſpicere quæſierit, triangulum ACM à triangulo MDL ſubducat hoc modo, à linea MD, linea MC præcidat, & à linea ML lineam MA obtruncet, & lineam MN ducat, triangulum MIN æquale erit ACM, reliquum triangulum IDN erit quantitas lunulæ IDML, dempla lunula AEC.

Vacua inter lunulas intermiſſa quadrare.

Prop. 5.

AT vero interuenientia vacua circa lunulas ſi quadrare quæſieris, ita quadrabis. Eſto minor lunula ABCG, maior CDLH imperamus ſemicuruilinea triangula inania inter illas, in rectilineas figuras reddere ſcilicet CPD, DQL, CHL, ARB, BSC, AGC, circumſcribantur parallelogrammata tangentia earum ambientes lineas PQCL, ARSC, & fiat alterum parallelogrammum ex binis AL, TV, & ſit ALTV, & fiat triangulum ACL æquale AFL (per 31. primi Euclid.) quibus ita diſpoſitis inquam vacuum circa ATVLF æquale eſſe imperatis vacuis.

cuis. Quoniam triangulum A F L eſt æquale A C L ex

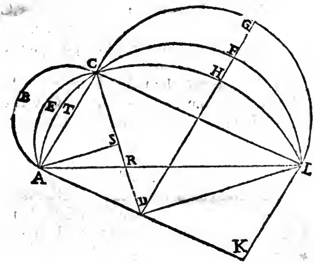


conſtitutione,& triangulum ACL eſt æquale lunulis CDLH, & A B C G, ergo ſi triangulum A F L à parallelogrammo A T L V abſtuleris, reliquum vacuum A T V L F erit æquale interiectis vacuis iam recenſitis.

Duas lunulas inæquales in ſemicirculi ambitu deſcriptas ſeorſum quadrare.

Prop. 6.

ESTO rectangulum triangulum A C L, cuius porrectius latus C L ſit duplum exilioris AC, & circumferantur lunulę ex more,quibus adijce ſuas literas indices C G L F, & A B C T, mox parallelogrammum conſtituatur ex lateribus



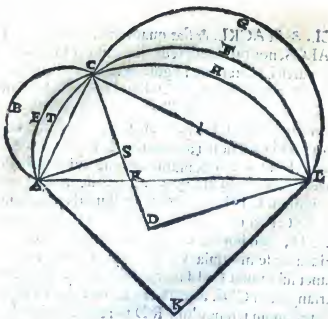
ribus AC, CL, & ſit ACKL, & fiat quadrans circuli CDLH, & ſubdupli AECS, nos rationem reddituri, lineam CR triangulum ACL partiri taliter, vt anguli compares mutuo correſpondentes, & æquales ſint, vt ACR par ſit RDL & triangulum A C R par ſit lunulæ A B C T, & triangulum CRL ipſi CGLF. Quoniam lunulæ CGLF, & ABCT pares ſunt triangulo ACL quarta huius noſtri adiuuante, & triangulo ACL par trimetrum C D L, quoniam vtrique ſui parallelogrammi dimidium eſt (vt figura quarta huius demonſtratum eſt) ergo triangulum C D L eſt duabus præſignatis iam lunulis CGLF,& ABCT æquale, ſed triangulum CDL eſt æquale lunulæ C G L H, ergo lunula CGLH eſt æqualis CGLF, & ABCT ſubducatur ſemilunula CGLF, vtpote vtrique communis, remanet ſublunula CFLH æqualis ABCT, & quemadmodum triangulum CDL æquale ACL, ſubducatur commune C R L, reliquum triangulum R D L reliquo triangulo ACR æquale, lunularũ partium repreſentantia, ſequitur trian-

K gulum

gulum RDL eſſe æquale ſublunulæ C F L H, & triangulum ACR æquale lunulæ ABCT, à quo ſi triangulum ACS ſubtrahatur, æquale lunulæ ABCE (per primam huius) remanet ſubtriangulum ASR imæ lunulæ A E C T par, quod erat demonſtrandum.

Vel hoc modo ſi triangulum A C L æquale eſt lunulis A B C T, C G L F, compleatur triangulum C R L, & compleatur perfecta lunula, quę ſit CGLH, ergo addita pars trianguli RDL æqualis erit additæ lunulæ CFLH, ſed pars trianguli addita RDL æqualis eſt triangulo A C R, vt vidimus, & par eſt lunula ABCT lunulæ CFLH.

Vel hoc modo. Tres lunulæ ABCE, CGLF, AECT ſunt æquales trigono ACL, & duæ lunulæ CGLF, CFLH trigono C D L, ergo omnes quatuor iam dictæ lunulæ ſunt æquales duobus trigonis A C R, & C D L, ſed tres lunulę A B C E,



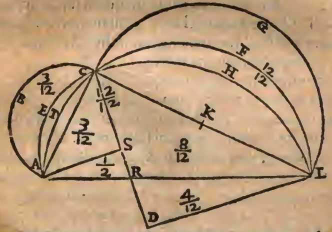
C G L F, L F C H ſunt æquales trigono ALK, ergo reliqua

lunu-

lunula A E C T eſt æqualis trigono A S R, quod quęrebamus.

Vel tres lunulę A B C E, AECT, CGFL æquales ſunt trigono ACL, & trigonum CDL æquale lunulę CGLH, tolle triangulum CDL ęquale iam dictę lunulę CGLH, reliquum triangulum ACR lunulis ABCE, AECT æquale eſt, tolle lunulam perfectam A B C E ſub lunula A E C T ſub triangulo A S R æqualis erit, quod erat demonſtrandum.

Trianguli in circulo deſcripti angulo per medium diſciſſo, & lunulis à circulo medio diuiſis, proportio partis maioris trianguli ad minorem, eſt ſicut ſuperior pars maioris lunulæ ad inferiorem, & ſuperior eadem pars ad minorem lunulam, & ſuperior pars trianguli ad inferiorem ſequitur eam ſuarum lunularum. Prop. 7.

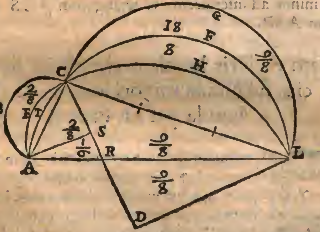


Priuſquam ad diuerſarum partium rationem lunularum

deſcendamus, admonitione dignum cenſemus, quod cum ab æqualitate duarum lunularum deſcendimus, quam (in ſecunda parte) vidimus, quantum maior creſcit, tantum altera decreſcit, & ex alterius defectione altera augmentũ ſuſcipit, & circulus ille, qui per medium vtriuſque percurrit, à maiori ſemicirculo ſubripit, & minori addit, ſed id non temerè, ſed certo ſe ſuperant exceſſu, vt ratio maioris ſuperioris lunulę ad inferiorem eadem ſit, quam maioris ſuperioris trianguli pars ad inferiorem, & ratio maioris ſuperioris lunulę ad totam minorem, vt ratio partis trianguli anterioris ad poſteriorem eandem ſequuntur an. log am, & vtraque vtriuſq; rationem ſequitur, vt exemplis patebit. Triangulum rectangulum ſtrue, cuius angulis appinges literas ACL, productius latus CL in duas partes, anguſtius in vnam partiri. In puncto bifariæ ſciſſionis lateris C L ſigna K, ex quo, & interuallo CK circinationis arcus exaretur, cui ſuas indices literas applicabis C G L, eodemq. ordine ſigna A C ſuum arcum delinea ABC, mox triangulum maioris circuli CGL conſtitues, & ſit C D L, & minoris ABCE, ſit ACS ſupra ſuccumbentem omnium baſim AL, medius circulus flectatur ACFL, dico lunulam ſuperiorem maioris circuli C G L F ad ſuam inferiorem CFLH, eandem habere rationem, quam ſuperior pars trianguli CRL ad inferiorem RDL, & eadem ſuperior pars lunulę maioris CGLF ad totam minorem ABCT, quam triangulum CRL ad ſuum ſequentem ACR.

Quod vt facilius cognoſcamus ad hoc demonſtrandum adhibeo numerorum officium, & vt facilius proportiones obſeruemus cum fractis integros numeros in fractiones ſoluamus, vt vnum denominatorem habeamus. Quoniam lineam C L in duas partes diuiſimus erit eius quadratum quatuor partium, cuius pars quarta, ideſt vnitas eſt, hanc in duodecimas ſoluemus, ideſt $\frac{12}{12}$, erit ergo totum triangulum ACL$\frac{12}{12}$, & quia linea C L ad CA, duplam habet proportionem, ita RL ad RA, & ſic triangulum CRL ad triangulum CAR, ergo trian-

triangulum CRL erit $\frac{8}{12}$, & triangulum ACR $\frac{4}{12}$. Ergo tota lunula perfecta CGLH $\frac{12}{12}$ erit ſuperior lunula maioris circuli CGLF cum duabus lunulis ABCT ſimul iunctæ ſunt $\frac{12}{12}$. Triágulum CDL erat $\frac{12}{12}$, ſi ſuperior pars erat $\frac{8}{12}$ inferior RDL erit $\frac{4}{12}$, & quia tota perfecta lunula CGLH eſt $\frac{12}{12}$; vt proportio quadrupli ſeruetur, quam habet ad minorem perfectam ABCE $\frac{3}{12}$, & triangulum ACS erit ei par $\frac{3}{12}$, inferior ergo pars $\frac{1}{12}$, & cum nullus melior congruat ſublunularum partibus ſit ſublunula AECT $\frac{2}{12}$, ſic ſuperior pars maioris lunulæ $\frac{8}{12}$ inferior $\frac{4}{12}$. Vnde proportio trianguli ſuperioris CRL ad inferiorem BDL erit dupla, vt ſuperior maioris lunulæ pars CGLH ad inferiorem etiam dupla, ideſt $\frac{8}{12}$ ad $\frac{4}{12}$, & proportio ſuperioris lunulæ pars CGLF ad lunulas ABCT etiam dupla eſt, ſed ſuperior minor lunula ad inferiorem tripla eſt, vt $\frac{3}{12}$ ad $\frac{1}{12}$, ſic triangulum ſuperius ACS ad ASR, vt $\frac{3}{12}$ ad $\frac{1}{12}$.



Subijciatur aliud exemplum in tripla diuiſione ſupra lineam porrectam AL, circulum effingamus ACFL, mox in eo triangulum deſcribendum, vt rectus angulus C literam poſſideat, cuius latus vnum longius CL tres partes, minus AC, vnam ſortiatur in ſtruendis ibi commodè ſemicirculis CGL

oi maior,

maior, ABC minor. Inde perfectæ lunulæ ex ordine ſubſignentur, ideſt trianguli CDL, CGLH, trianguli ACS, ABCE.

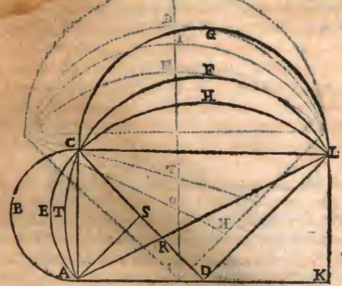
Triangulum ACL, quia linea eſt trium partium, area erit $\frac{12}{8}$, proportio CRL ad ACR eſt tripla, ob id triangulum CRL erit $\frac{9}{8}$, & triangulum ACR $\frac{3}{8}$, triangulum CDE latus trium eſt partium, quadrans eſt nouem partium, cuius quartæ pars eſt $2\frac{2}{8}$, ideſt $\frac{18}{8}$ ſupra ſunt trianguli CRL $\frac{9}{8}$, ergo triangulum RDL erit $\frac{9}{8}$, tres lunulæ ABCE, AECT, CGLF eſt $\frac{12}{8}$ ſuperior maior ad duas minores eſt tripla, ergo lunula ſuperior eſt $\frac{9}{8}$, & duæ lunulæ ABCE, AECT eſt $\frac{3}{8}$, ſed lunula perfecta eſt $\frac{3}{8}$, ergo ſuperius triangulum CAS $\frac{3}{8}$ ſubditum reliqui erit $\frac{3}{8}$ & ſublunula reliqua $\frac{3}{8}$ inferior maior lunula, vt compleat numerum $\frac{12}{8}$ erit $\frac{9}{8}$, ergo proportio lunula ſuperior ad inferiorem, ideſt CGLF ad CFLH ſicut triangulum CRL ad RLD, & lunula EGLF ad lunulas ABCT, & ſuperior lunula minor ad inferiorem, vt triangulum ACS ad triangulum ASR.

Datam maioris lunulam circuli ita ſecare, vt eius ſublunula minori lunulæ, & ſublunulæ par ſit. Prop. 8.

Qvamquam in ſupra commonitis idem indicauerimus, vberioris tamen doctrinæ gratia exemplum in hunc modum abſoluemus. in hanc rationem triangulum orthogonium eligendum eſt ACL, cuius porrectius latus CL bifariam diſpeſcemus, ſupra lunulas ex more conſtituemus, & medium circulum ATCFL, & eis triangula ſubijciemus CDL, ACR, mox datis lineis AC, CL parallelogrammum conſtituatur CALK, inquam ſublunulam CFLH, lunulis ABCT æqualem eſſe. Quoniam lunula CGLH æqualis eſt ſuo triangu-

lo

lo CDL, & triangulum CDL æquale triangulo CAL, quia



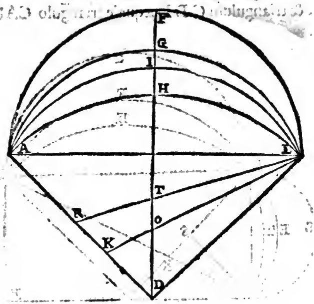
vtrumque eſt æquale ſuo parallelogrammo dimidio CALK, & triangulum ACL eſt æquale lunulæ CGLF, & lunulis ABCT, ergo lunula CFLH eſt æqualis lunulis ABCE, & AECT, tollatur de medio lunula CGLF, remanebit ſublunula CFLH æqualis ABCT, quod erat diſcutiendum.

Datam dupli, & ſubdupli circuli lunulam ad imperatas partes diſcindere.

Prop. 9.

Eſto dupli, & ſubdupli multifariam ſcindenda lunula AFLH, cuius ſubtenſa AL, ſui compar triangulum inferius ſubſtituemus ADL à medio inflexi circuli vertice F vſq; ad trianguli aciem infernè poſitam D recto ductu deſcendat. In dextro trianguli latere AD futuræ lineæ deſignandæ ſunt lu-

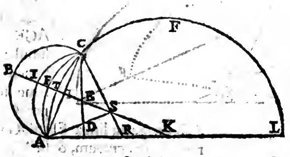
nulam



nulam diuiſuræ in quotcunque partes volueris decem, vel ſeptemdecim, pro nunc bifariam in R diſpeſcito, & tranſuerſam ad L deducito, ſi velis lunulæ mediam partem auferre, & vbi ſe lineæ cum diametri linea decuſſabunt, fige circini pedem fixum, & vagum alterum ad alterutrius diametri extremitatem A, vel L, arcum circumflecte A G L, nam lunula AFLG erit dimidium lunulæ. Vel ſi tertiam partem vis auferre, ſit latera K in laterali tractu tertia pars ab K ad L lineam porrigito, & vbi FD lineam ſecat, pede circini ſtabili collocato, ac vago altero ad A extremum, arcum circumduces A I L, & tertiam lunulæ partem A I LH à duobus abſcindet. Quoniam triangulum ADL per RL lineam bifariam diſſectum eſt, ſuperior trianguli pars ARL per ſuperexaratam propoſitionem ſuperiori lunulæ parti AFLG congruit, & ima trianguli pars RDL imæ lunulæ parti correſpondet, & vtræq. partes ſunt, ergo & lunulæ eis pares diſſectæ ſunt: Idem de tertia parte dicendum.

Eadem

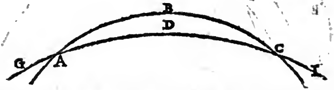
Eadem in parua lunula operaberis, nam ſi bifariam ABC lunulam vis partiri trianguli eius ABS latus, AS bifariam in D, & latus à D quouſque ad punctum C perueniat protrahe-



mus, & lineam à B medio ſemicirculi, punctoq. S, vſque ad circuli A T C F L centrum perueniat, & vbi ſe interſecabunt puncto E ſiſte circini pedem, & Interuallo EA, extende circulū AIC, & diuiſa erit lunula: probatio ex anteriori liquet.

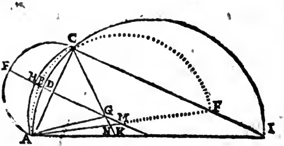
Datam quamcumque lunulam quadrare. Prop. 10.

ESto expoſita quadranda lunula A B C D cuiuſcunque ordinis, cui æquale oportet reperiri rectilineū, maior cir-



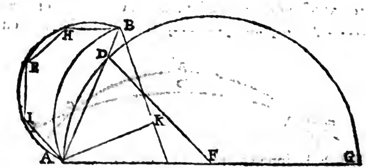
culus GADCH, integer circinetur, & ſit ADCI porrecto ſuo diametro A I, & à puncto A ſuperponatur præfata lunula ABC, & compleatur circulus A B C F cum ſuo diametro AF, extendaturq. linea à puncto A ad C, & ſit AC, diſcindaturq. rectus angulus ACI per rectam C K, & ſuper baſim AC circinetur ſemicirculus AEC, & puncto G, baſi AC, fiat quadrans

dupli,& ſit AHCG, & trahatur A G, G C. Quoniam triangulum A C G eſt æquale lunulæ ACEH, remanet ſubtriangulnm AGK æquale ſub lunulæ AHCD. à puncto ſemicircu li medio E, AEC per G, vſque ad



centrum circuli A D C I, dirigatur linea EHBDGML ex circumferentia A B C reperiatur centrum, & ſit M, & trahatur A M, vſque ad F diametri finem ex ſupradictis, ſi à triangnlo AGK, quod lunulam AHCD refert, ſubducatur AGM, quod repręſentat lunulam A H C B, remanet ſubtriangulum ANK repręſentans ſublunulam ABCD, quod erat edocendum.

Semilunulas ex nota ratione quadrare. Prop. 11.



IN ſola proportione dupli, & ſubdupli accidit, vt ſemidiameter circuli ſubdupli ſit æqualis ſubtenſæ quadrantis dupli, vt in prima Prop. huius vidimus: ideo in his perfectæ lunulæ, & alijs potius circuli ſemilunulæ dici poſſunt.

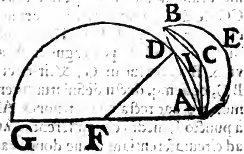
Eſto

Eſto circulus ADG quadrans ſemicirculi AEB, & quarta pars ſemicirculi ADG ſit ADF; & diameter ſubdupli ADB tranſeat per punctum D, dico triquetrum ADF ſemicirculo AEDB æquale eſſe. Tollatur communis portio DA, remanet lunula AEBD, æqualis triangulo FDA, ſed AEBL perfecta lunula nota eſt: ergo ſublunula ALBD nota erit, ſubducito triangulum DBK ab AKF, reſiduum ſublunulæ ALDB notum erit.

Poteſt & alio modo cognoſci. Portio quadrupli AD valet quatuor portiones ſubquadrupli, ſecetur circumferentia AEB quadrifariam, & ſint portiones AI, IE, EH, HB, amputentur portiones AI, IE, EH, HB, reponatur AB, trapezium rectilineum AIEHBD notum erit, & ſic de cæteris.

Alio modo.

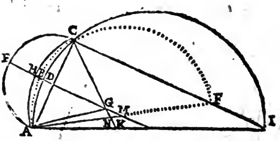
Sit ſemicirculus ADG, quadruplus ipſius AEB, & ipſius AEB, capiatur duplus, & ſit circumferentia ACB, remaneat infra ſublunula ACBDIA, quam volumus quadrare. Quia ſemilunula ACBDIA, nota eſt, &



nota etiam lunula AEBC, ex vigeſima ſexta primi noſtri, ſi notum à noto ſubtrahatur, quod reliquum eſt notum erit.

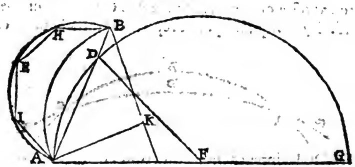
Poſſumus etiam proximè prædicto modo quadrare, quia portio AD, eſt dupla ipſius ACB, diuidatur ACB, in duas partes AC, CB, demantur hæ duæ portiones AC, CB, reponatur AD, rectilineum ACBDA, eſt æquale ſublunulæ ACBDIA.

dupli,& ſit AHCG, & trahatur A G, G C. Quoniam triangulum A C G eſt æquale lunulæ ACEH, remanet ſubtriangulnm AGK æquale ſub lunulæ AHCD. à puncto ſemicircu li medio E, AEC per G, vſque ad



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Semilunulas ex nota ratione quadrare. Prop. *11*.



IN ſola proportione dupli, & ſubdupli accidit,vt ſemidiameter circuli ſubdupli ſit æqualis ſubtenſæ quadrantis dupli, vt in prima Prop. huius vidimus: ideo in his perfectæ lunulæ, & alijs potius circuli ſemilunulæ dici poſſunt.

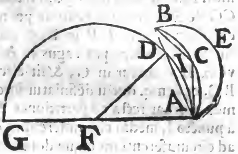
Eſto

Eſto circulus ADG quadrans ſemicirculi AEB, & quarta pars ſemicirculi A D G ſit A D F, & diameter ſubdupli ADB tranſeat per punctum D, dico triquetrum A D F ſemicirculo AEDB æquale eſſe. Tollatur communis portio DA, remanet lunula AEBD, æqualis triangulo FDA, ſed AEBL perfecta lunula nota eſt: ergo ſublunula A L B D nota erit, ſubducito triangulum D B K ab A K F, reſiduum ſublunulæ A L D B notum erit.

Poteſt & alio modo cognoſci. Portio quadrupli AD valet quatuor portiones ſubquadrupli, ſecetur circumferentia AEB quadrifariam, & ſint portiones AI, IE, EH, HB, amputentur portiones AI, IE, EH, HB, reponatur AB, trapezium rectilineum AIEHBD notum erit, & ſic de cæteris.

Alio modo.

Sit ſemicirculus A D G, quadruplus ipſius A E B, & ipſius AEB, capiatur duplus, & ſit circumferentia ACB, remaneat infra ſublunula A C B D I A, quam volumus quadrare. Quia ſemilunula ACBDIA, nota eſt, &

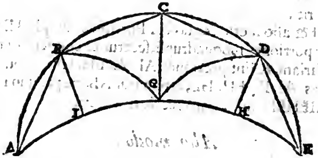


nota etiam lunula AEBC, ex vigeſima ſexta primi noſtri, ſi notum à noto ſubtrahatur, quod reliquum eſt notum erit.

Poſſumus etiam proximè prædicto modo quadrare, quia portio A D, eſt dupla ipſius A C B, diuidatur A C B, in duas partes AC, CB, demantur hæ duæ portiones AC, CB, reponatur A D, rectilineum A C B D A, eſt æqualæ ſublunulæ A C B D I A.

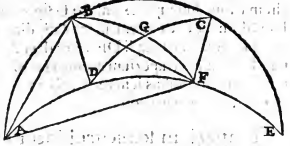
Lunulam per latum in quadrabiles partes diuidere. Prop. 12.

NNunc me conuertam ad quadrandas lunularum in latum partes, cum paulò ante per longum indicaueri-

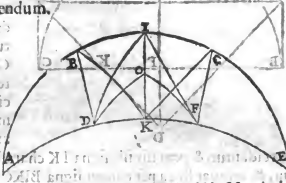


mus. Eſto lunula AECG, diuidaturq. per medium per lineam CG, & AC circumferentiam per medium proſcindes in B, & ex aduerſa parte CE in D, & inuariata circini apertura ſemicirculi ſubdupli pes vagus in A, alter fixus ſiſtatur taliter, vt ex B voluatur in G, & ſit circumferentiæ circumductio B G, eodemq. ductu definiatur intercapedo GD ſimilis BG, mox prorúat recta ad partitiones AB, BC, CD, AG, GE, & à puncto I, medio circumferentiæ AG, erige lineam ſurſum ad circumferentiam vſque donec eam in B tetigerit. Quoniam A B eſt æqualis B G, & portio A G dupli rependit duas AB, BG ſemidupli, dico illis AB, BG ſublatis, hac AG repoſita, rectilineum triangulum A B G notum erit, idem intelligendum de altera partitione GDE, reliquum triangulum BCDG notum erit, vt reſiduum notæ lunulæ. Vel amputatis portionibus BC, CD, repoſitis his BG, GD æquales, & eiuſdem circumferentiæ, recenſitum triangulum notum erit. Ex his triangula ABG, BCDG, GDE per medium tranſuerſis diuiſa lineis nota erunt, & tripartita erit lunula.

Vel



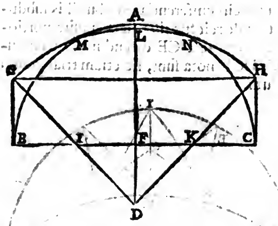
Vel circuli dupli tripartita partitione curuaturā conſtituas A D, DF, FE; Item ſubdupli AB,BC,CE; inuariataq. circini apertura intra duo puncta B F, DC mutuo inter ſe intercidentes arcus flectantur BF, DC, & vbi futurus eſt cōtactus illic pone G, aio triangulum ABF notum eſſe, quia AF, dupli circumferentia, duobus illis ſubdupli AB, BF reſpondet; vnde reiectis illis, hac repoſita portione penſatur. Idem de alia parte DCE dicendum, & triangula BGD, CGF, BGC, DGF nota ſunt, ſic etiam triangulum BCF, & FCE dicendum.



Remanente adhuc lunulę illæſa tripartitione ex puncto D fixo circini pede altero ad F ſigna in ſubduplo æquale portionē in I, mox inuariata circini apertura, qua duplū circulū conſtituiſti, ſiſte circini pedem in altera extremitate lunulæ puta E, alterum ad D, & I conuertè, & ſigna circumferentiam dupli DI, & ab I ad F excurrat recta I F, dico triangulum DIF notum, quia circumferentia dupli DF eſt æqualis DI ex conſtitutione, ratione iam ſæpius repetita, addendo, & demendo notum erit triangulum DIF, vel intercapedine BK circinetur BK circumferentia dupli, & recta ducatur K C, quia DF, eſt compar BC, & DF æqualis ipſi BK, tolle portionem BC, repone BK triangulum

lum BKC notum erit, vel deducta linea IK, & interuallo B I circinatione ſubdupli ex puncto D ſigna in linea IK interuallum BI in O, & ex altera parte OF, dico triangulum DOF notum, quia linea dupli DF valet illas duas DO, OF ſubdupli, & reliquum trapezium notum erit BDOFC demendo notum à noto, ſimilibus iam recenſitis modis multipliciter perlatum lunula partiri poterit.

Trapezium ſemicuruilineum trapezio par extruere. Prop. 13.



Eſto dupli quadrans DGLH, & hinc inde à terminis G H tranſuerſa linea ducenda eſt, quæ curuaturam ſpectet, & ſit GH, mox duo latera, quæ anguli faciem in D confirmant DG, DH, amuſſim in medio præcidantur, & præciſionis ſigna I K characteribus inſigniantur, & proruat linea per eadem ſigna BIKC ipſi GH paralleſa, & ex punctis G H ad perpendiculum demittantur ſupra B C lineæ G B, G H, & excurrentes in longum lineam BIKC bifariam diuidat, cuius diuiſionis terminus F, & centro F interuallo F B, circumducatur ſemicirculi forma ſubdupli B A C, quæ quadrantis circumferentiam in duobus punctis intercidet. Interciſionis puncta totidem literas ſortiantur M, N; & vniatur F D. Dico Trapezium ſemicuruilineum BGMLNHC par eſſe quadranti DGLH, ſed ad demonſtrationem accingamur. Quoniam F D æquidiſtat H C, cadit

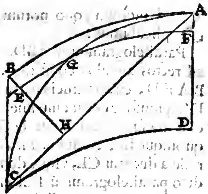
inter

inter eas D H, ergo anguli F D K, K B C æquales anguli ad K contrapoſiti etiam pares : Itidem & recti ad F C, & H K li‑nea ipſi KD etiam æqualis. Triangulum ergo FDK triangulo K H C compar. Idem ex contraria parte ſentiendum, cum compari linearum deſcriptione confirmata ſit. Demantur ergo triangula IFD, FDK reponantur GBI, KHC, trapezium ſemicuruilineum B G M L N H C æquipollet ſemicirculo B M A N C.

Conſectarium.

Ex his apparet lunulam MANL æqualem eſſe duobus triangulis MGB, NHC, quoniam circuli pars extramittitur MANL, includantur trapezij partes NHC, MGH.

Triangulum ſemicuruilieum ex quarta ſemicirculi ſubdupli, & octaua dupli quadrare. Prop. 14.



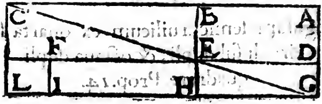
Sit ſemiabſciſſa lunula AGCD, & linea CB ipſi AD parallela conſtituatur, & dupli circumferentia A B extremitate A ipſi CD parallela ducatur, donec ductum lineæ CB contingat in B, dico ſemicuruilineum triangulum ABCG quadrari poſſe. Quoniam AB ipſi CD Parallela, & eiuſdem circumferentiæ, ergo quadrangulum BACD notum eſt, à quo ſi ſemilunula ſubducatur CGAD, remanet triangulum notum ABCG.

Vel

Vel quia lunula æqualis eſt AGF ipſi ECG ex præcedenti, parallelogrammum EBAF eſt notum, quia ex æqualibus circumferentijs; ergo & triangulum CBA notum eſt, quia vtrique additur commune GAF.

Vel triangulum ſemicuruilineum, quod paulò ante deſcripſimus ABCE remaneat,& ducatur linea CA portio ſemidupli CGA, & ſit ABH ſemiportio dupli, ergo æquales, abſceſſus communis EHGA tollatur, remanet BEA æquale CEH, addatur vtrique communis BEC, ergo triangulum rectilineum BHC eſt æquale triangulo ſemicuruilineo BCGA.

Notum à noto ſubtrahere. Prop. 15.



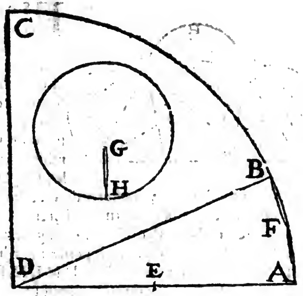
Sed modum, quo notum à noto ſubtrahatur nunc explicabimus.

Parallelogrammum ABDE circa decuſſatas lineas BH, EF ad rectos angulos conſtitue, & ſit quantitas lunulæ datæ BAED, ex alia crucis parte HIEF, elongeturq. parallela IH, quouſque coeat cum linea AD in puncto G, mox linea GE diſcindens angulum DGH, ducatur per coniunctum E, quouſque intercidat lineam AB,& ibi appone C, & ab C, parallela ducatur CL, vſque donec lineam GHI tetigerit in L, dico parallelogrammū FLI eſſe quantitatem trianguli BAC, qua ſuperatur à parallelogrammo FH, quoniam parallelogrammum BD eſt æquale parallelogrammo EL, dempto parallelogrammo FH, reſiduum erit FL, quod quærimus.

Circu-

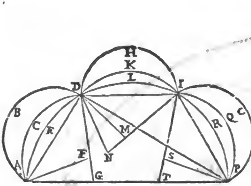
Circulum quadrato proximum conſtituere.

Prop. 16.



ABſoluta lunularum tractatione venio tandem ad circuli quadrationem ex ingenij facultate, ſed initio circuli quadrationi approximabimur. Eſto ſexto decupli circuli quadrans A B C, & ex ſemidiametri dimidio D E, ſub ſexto decuplus circulus conſtituatur GH, amputetur pars quadrantis ADC, ſitq. DBA, dico triangulum DBA circulo mutua parilitate conſtare ſecetur arcus quadrantis BA bifariam in F, connectanturq. recte BF, FA, & à circulo G H itidem duæ illæ portiones quadrantis BF, FA, dico triangula FDB, ADF eſſe æqualia circulo, demptis duobus portionibus GH, æquales illis BF, FA interclufis: quod patet ex conſtructione.

Datum circulum quadrare. Prop. 17.



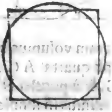
ESto expoſitus circulus A D I P, inuariataq. circini apertura ſignētur in circuli circumferentia tres abſceſſus tres. n. recipiet compares,& coæquales ſibi correſpondentes ſemidiametro ſemicirculos (cogēte ad id Euclidis Propoſ.15.4.) quibus applicentur diametri AD, DI, IP, ſupra quorum centra incuruentur ſemicirculi ABD,DHI,IOP, excurrat linea ex D ad P, diuidaturq. angulus A D P per lineam DFG, & ex arte ſuperius repetita fiant lunulæ ABDC, DHIK,IOPQ, & triangula A D F, D M I, I S P cum ſuis ſub triangulis ſuis ſublunulis correſpondentibus A F G, D N M, P S T.

Quoniam ſemicirculus A L P conſtat quatuor ſemicirculis æqualibus ſemidiametro, ſi tollantur tres portiones communes A E D, D L I, I R P, & tria triangula æqualia lunulis AFD, DMI, ISP, tollanturq. tres ſublunulæ tribus ſubtriangulis reſpondentes ACDE, DKIL, IQPR, cum ſuis ſubtriangulis reſpondentibus illis AFG,NDM,SPT,vacuum reliquum intercedens rectilineum, vel trapezium G D N M I S T valet quantum ſemicirculus quartus relictus X Y Z, hoc inane valet ſemicirculus X Y Z. Abſoluamus igitur circulum cum ſuo quadrato valente trapezium illud, & quadratus erit circulus.

Nunc

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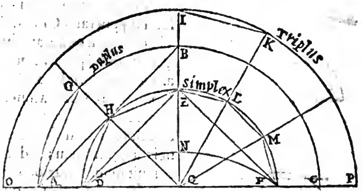


Nunc euertatur, & faceſſat Hyppocratis Chi; fallacia circulum quadrare satagentis, quod putarat quemadmodum lunulæ dupli, & ſubdupli in quadratum adducebantur, ita quamcumq. circulorum cum ſuis rectilineis æquationem; ſed eius corruit demonſtratio, nam res ſe aliter habet: quod enim eſt ſingulare in circulis ſe in dupla proportione excreſcentibus, diſſentaneum eſt idem in reliquis exiſtimare. Nos (ni fallimur) ex inuentione trianguli A F G ſublunulæ aereæ A D C E reſpondentis aſſecuti ſumus.

Data portione nota, alteram cuiuſq. proportionis ſibi comparem perueſtigare. Prop. 18.

ESto linea O P futura baſis variorum circulorum, & ſemicirculus O I P triplus, A B C duplus, & D E F ſimplex, ſiue ſubduplus, & ſic de alijs alter ſupra alterum in quæſita ratione ſemper excreſcens, & à puncto Q, quod medium diametri poſſidet angulis vtrinque æqualibus aſcendat linea Q I ſemicirculus bifaria diuiſione diſcendens: mox ex centro Q circumferentiam AB æqualiter partiens vſque ad G, tranſmittatur, ex altera parte duæ aliæ circumferentiā I P, trifariam ſecantes coæquentur, & ſint Q K, Q M, & ſit

 data



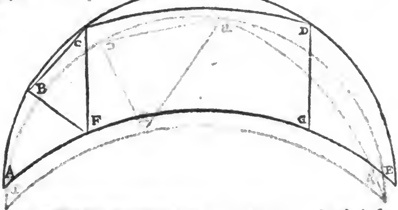
data ſimplicis circuli portio DE, alteram volumus in dupla proportione æqualem inuenire. Dupli quarta A G B diſcindatur bifariam, trahaturq. linea AG, & à puncto D ad E dupli portio ſubpingatur. Quoniam vidimus in lunula DEFN, portionem dupli quartæ circuli DNF valere duas quartas circuli ſubdupli, vel ſimplicis D H E, E L F, ſed portio dupli D N F eſt æqualis A G B, quia eadem eſt dupli quarta. Ergo area concluſa in portione dupli DNF valet duas quartas ſubdupli DHE, ELF, & portio octaua dupli AG valet duas octauas ſubdupli D H, H E. Idem dicendum de tripla, nam circulus O I K P eſt triplus ſubtripli, vel ſimplicis circuli DHELF, & par ſexta ſemicirculi tripli I K, valet tres ſextas ſubtripli circuli EL, LM, MF, & ſic de alijs cuiuſcunque quantitatis, & incognitæ menſuræ dicendum: nam ſemper rata, & iuſta proportio erit.

Ex portionibus circulum quadrare. Prop. 19.

Ex ea, quam modo expoſuimus propoſitione dependet hæc portionum quadratio, quam ob oculos exponemus.

Eſto propoſita lunula A B C D E, ex qua (docente id 15. propoſ. noſtri ante præteriti) minor lunula abſcindatur,

quæ

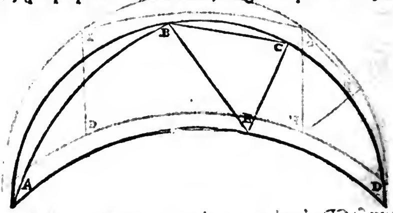


quæ ſit CD, ab cuius extremitatum terminis infernè deſcendant perpendiculares CF, DG. Quoniam quadrangulum ſemicuruilineum CFDG notum eſt ex eiſdem æqualibus circumferentijs dupli CD, GF, nota quoque eſt minor lunula CD, ſi à lunula ACDE ſubducatur quadrangulum notum CDGF, & lunula CD reſidua cornicula ACF, DGE nota erunt. Mox lineæ dupli AF ſubdupli compar AB reperiatur (per propoſitionem proximè præteritam) & erit trilaterum BFC, quod ſubtrahes à corniculo ABCF, & erit triangulum BCF notum. Duc lineam BC, & trilaterum rectilineum BCF notum ſubtrahe à ſemicuruilineo BCD noto, & area intra portionem BC concepta nota reſultabit.

Altera.

ALiam figuram extruendam non putamus, ſed ſuperiore lunula relicta. Eſto à puncto A vſque ad B duo puncta, dupli circumferentia flectatur, & ſit lunula minor AB, & æqualis AE, fiat linea AE dupli, & à puncto B ad E rectus

trames ducatur. mox lineæ ED compar ſubdupli reperiatur CD, (per propoſit. 2. noſtri huius) iunganturq. lineæ EC, &

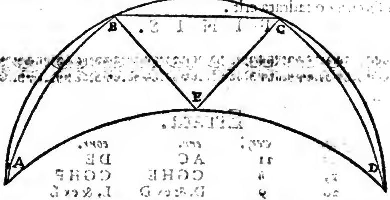


BC, recta lineam etiam connectatur. Quoniam AB, AE æquales, & eædem circumferentiæ ſunt, ſi lineæ ſubtendantur arcubus AB, AE amputata portione AB, repoſita ſuo loco AE notum erit triangulum ſemicuruilineum ABE, lunula AB nota eſt (ex propoſ. 15. præſentis noſtri) ergo lunulæ pars ABE nota erit, deme AB integram, innoteſcet reſiduum BCDE lineæ ED comparem reddidimus CD. Ergo corniculus ECD notus erit, deme à toto reſiduo BCDE innoteſcet trimetrum ſemicuruilineum BEC, appinge lineam BC à terminis BC, & triangulum rectilineum BCE notum erit, ſubducito à curuilineo, & portio BC nota erit.

Altera.

VEl in lunula ABCDE à puncto A vſque ad E, nota in ſubduplo, & ſit AB, & ex altera parte CD, & à puncto A, vſque ad B appinge lineam dupli AB, & ex altera parte CD, mox connectè lineas rectas BE, EC, BC, quia AB, AE æquales ſunt, ſic CD, DE, ergo lunulæ AB, CD etiam notæ, tolle, quia remanet par medij BEC nota, à noto

to ſemicurvilineo BCE, tolle rectilineum BCE, remanet no-

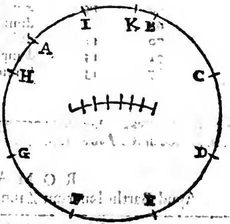


ta portio BC, ſic & aliæ diuiſiones imaginari poſſunt, & portiones quadrari.

Data vna portione nota, totam circumferentiam circuli quadrare.

Prop. 20.

SIt propoſitus circulus ABCDEFGH, & ſit portio nota AB, nos hanc circumferentiam metiemur ſepties AB, BC, CD, DE, EF, FG, GH, ſupererit HA, qua metiemur bis AB id eſt AI, IK, & ſupererit KB, quæ portionem IK ter menſurabit. Ergo AB ſepties partiemur, quibus abſciſſionibus ſuas partes addemmus. A portione igitur AB ſubducemus octogonum



num rectilineum, reliquum in ſeptem portiones diuidemus, ex quibus portiones tres recipiemus pro HI, & ſic tota circumferentia quadrata erit.

FINIS.

Errata.

pag.	verſ.	err.	corr.
11	21	AC	DE
13	8	CGHE	CGHF
20	9	D, & ex D	L, & ex L
23	1	vtrunque	vtcunque
23	18	AFG	AFC
23	21	FG	FB
30	1	AD	AB
32	5	hit	hic
48	15	Prop 8.	Prop 7.
49	17	repeſitiſq.	repoſitiſq.
58	7	MNE	MNP
64	20	trapeſium	trapezium
65	8	duplis	duplicis
68	15	æquidiſtant	æquidiſtans
69	1	dimidium	quarta pars.
70	12	deſcriptum	deſcriptas.
71	15	dempla	dempta
87	13	Semicuruilieum	Semicuruilineum

ROMAE,
Apud Bartholomæum Zannettum. M. DC. X.

SVPERIORVM PERMISSV.